

8

Screws, Fasteners, and the Design of Nonpermanent Joints

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To meet a need, or an opportunity, the designer often has to conceive (or invent, draw up, or adapt) a form and express the connectivity of its parts in order to achieve a goal. Much of this mental sculpting is geometric in nature, because shape often permits the achievement of function. Just consider the one-piece can opener which creates a triangular aperture in a can top. This is a single body which, because of shape, completes a machine (the lever), letting the can provide the fulcrum, allowing a human to supply the energy to reliably cut and fold metal. Geometry and geometric thinking are always involved in formulating a mechanical concept.

In earlier times, casting, with all its freedom of shape, was important to the realization of desirable forms in metal. Limitations on the shapes and sizes that can be cast, however, restrict geometries. The idea of *joints* (an old one, too)—places where separate bodies could be assembled to create geometric forms not possible with casting—allows the designer more variation in shape, therefore more latitude with form. What would be the largest bridge if bridges could not be assembled from many parts, on site? Just as the mortise-and-tenon joint, and dowel pins, helped with wood structures, so did the nut and bolt and the rivet afford metal structures more freedom of form. The idea of a joint is not new, but it can still do much for you as a designer.

One kind of permanent joint is a nonpermanent joint that is never opened. In other words, a nonpermanent joint continues a built-in option: to disassemble or not. The descriptive phrase “nonpermanent joint” is useful only if you remember not to remove it from your arsenal of permanent joints.

The helical-thread screw was an important invention. It is the basis of threaded fasteners, and an important element in nonpermanent joints.

A threat to the function of a joint is the possibility that it opens unintentionally, thereby compromising its function, which is to preserve a greater, useful geometry. Strength and stresses due to load interplay under the control of the designer. If a designer cannot ensure the integrity of a joint, then its use is not available to achieve a greater geometric form, limiting conceptual opportunities. Hence we study joints first, to let our conceptual horizons broaden.

8-1 Thread Standards and Definitions

The terminology of screw threads, illustrated in Fig. 8-1, is explained as follows:

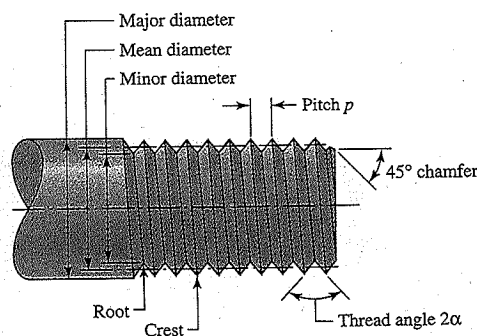
The *pitch* is the distance between adjacent thread forms measured parallel to the thread axis. The pitch in U.S. units is the reciprocal of the number of thread forms per inch N .

The *major diameter* d is the largest diameter of a screw thread.

The *minor diameter* d_r or d_1 is the smallest diameter of a screw thread.

Figure 8-1

Terminology of screw threads. Sharp vee threads shown for clarity; the crests and roots are actually flattened or rounded during the forming operation.



The lead l , not shown, is the distance the nut moves parallel to the screw axis when the nut is given one turn. For a single thread, as in Fig. 8-1, the lead is the same as the pitch.

A *multiple-threaded* product is one having two or more threads cut beside each other (imagine two or more strings wound side by side around a pencil). Standardized products such as screws, bolts, and nuts all have single threads; a *double-threaded* screw has a lead equal to twice the pitch, a *triple-threaded* screw has a lead equal to 3 times the pitch, and so on.

All threads are made according to the *right-hand rule* unless otherwise noted.

The *American National (Unified)* thread standard has been approved in this country and in Great Britain for use on all standard threaded products. The thread angle is 60° and the crests of the thread may be either flat or rounded.

Figure 8-2 shows the thread geometry of the metric M and MJ profiles. The M profile replaces the inch class and is the basic ISO 68 profile with 60° symmetric threads. The MJ profile has a rounded fillet at the root of the external thread and a larger minor diameter of both the internal and external threads. This profile is especially useful where high fatigue strength is required.

Tables 8-1 and 8-2 will be useful in specifying and designing threaded parts. Note that the thread size is specified by giving the pitch p for metric sizes and by giving the number of threads per inch N for the Unified sizes. The screw sizes in Table 8-2 with diameter under $\frac{1}{4}$ in are numbered or gauge sizes. The second column in Table 8-2 shows that a No. 8 screw has a nominal major diameter of 0.1640 in.

A great many tensile tests of threaded rods have shown that an unthreaded rod having a diameter equal to the mean of the pitch diameter and minor diameter will have the same tensile strength as the threaded rod. The area of this unthreaded rod is called the tensile-stress area A_t of the threaded rod; values of A_t are listed in both tables.

Two major Unified thread series are in common use: UN and UNR. The difference between these is simply that a root radius must be used in the UNR series. Because of reduced thread stress-concentration factors, UNR series threads have improved fatigue strengths. Unified threads are specified by stating the nominal major diameter, the number of threads per inch, and the thread series, for example, $\frac{5}{8}$ "-18 UNRF or 0.625"-18 UNRF.

Metric threads are specified by writing the diameter and pitch in millimeters, in that order. Thus, M12 \times 1.75 is a thread having a nominal major diameter of 12 mm and a

8-2

Thread profile for metric M
threads. D (d) = basic
diameter of internal
thread; D_1 (d_1) =
minor diameter of internal
thread; D_2 (d_2) =
pitch diameter of internal
thread; p = pitch; H
 $= 1.29944 p$.

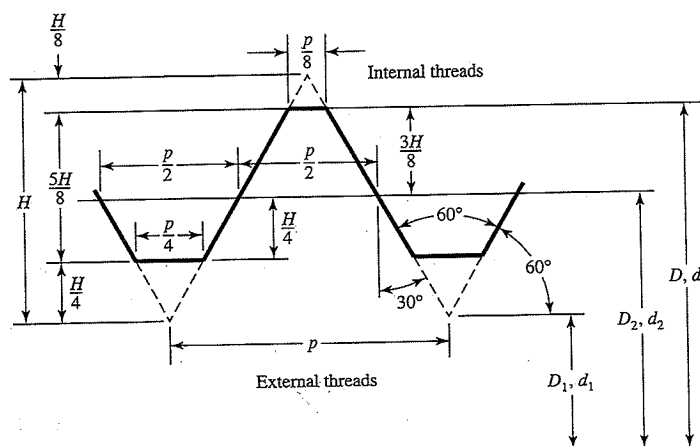


Table 8-1

Diameters and Areas of
Coarse-Pitch and
Fine-Pitch Metric Threads.
(All Dimensions in
Millimeters)*

Nominal Major Diameter d	Coarse-Pitch Series			Fine-Pitch Series		
	Pitch p	Tensile- Stress Area A_t	Minor- Diameter Area A_r	Pitch p	Tensile- Stress Area A_t	Minor- Diameter Area A_r
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

* The equations and data used to develop this table have been obtained from ANSI B1.1-1974 and B18.3.1-1978. The minor diameter was found from the equation $d_r = d - 1.226869p$, and the pitch diameter from $d_m = d - 0.649519p$. The mean of the pitch diameter and the minor diameter was used to compute the tensile-stress area.

pitch of 1.75 mm. Note that the letter M, which precedes the diameter, is the clue to the metric designation.

Square and Acme threads, shown in Fig. 8-3a and b, respectively, are used on screws when power is to be transmitted. Table 8-3 lists the preferred pitches for inch series Acme threads. However, other pitches can be and often are used, since the need for a standard for such threads is not great.

Modifications are frequently made to both Acme and square threads. For instance, the square thread is sometimes modified by cutting the space between the teeth so as to have an included thread angle of 10 to 15°. This is not difficult, since these threads are usually cut with a single-point tool anyhow; the modification retains most of the efficiency inherent in square threads and makes the cutting simpler. Acme threads are sometimes modified to a stub form by making the teeth shorter. This results in a larger minor diameter and a somewhat stronger screw.

Table 8-2

Dimensions and Area of Unified Screw Threads UNC and UNF*

Size Designation	Coarse Series—UNC				Fine Series—UNF		
	Nominal Major Diameter in	Threads per Inch N	Tensile-Stress Area A_t , in ²	Minor-Diameter Area A_r , in ²	Threads per Inch N	Tensile-Stress Area A_t , in ²	Minor-Diameter Area A_r , in ²
0	0.0600				80	0.001 80	0.001 51
1	0.0730	64	0.002 63	0.002 18	72	0.002 78	0.002 37
2	0.0860	56	0.003 70	0.003 10	64	0.003 94	0.003 39
3	0.0990	48	0.004 87	0.004 06	56	0.005 23	0.004 51
4	0.1120	40	0.006 04	0.004 96	48	0.006 61	0.005 66
5	0.1250	40	0.007 96	0.006 72	44	0.008 80	0.007 16
6	0.1380	32	0.009 09	0.007 45	40	0.010 15	0.008 74
8	0.1640	32	0.014 0	0.011 96	36	0.014 74	0.012 85
10	0.1900	24	0.017 5	0.014 50	32	0.020 0	0.017 5
12	0.2160	24	0.024 2	0.020 6	28	0.025 8	0.022 6
$\frac{1}{4}$	0.2500	20	0.031 8	0.026 9	28	0.036 4	0.032 6
$\frac{5}{16}$	0.3125	18	0.052 4	0.045 4	24	0.058 0	0.052 4
$\frac{3}{8}$	0.3750	16	0.077 5	0.067 8	24	0.087 8	0.080 9
$\frac{7}{16}$	0.4375	14	0.106 3	0.093 3	20	0.118 7	0.109 0
$\frac{1}{2}$	0.5000	13	0.141 9	0.125 7	20	0.159 9	0.148 6
$\frac{9}{16}$	0.5625	12	0.182	0.162	18	0.203	0.189
$\frac{5}{8}$	0.6250	11	0.226	0.202	18	0.256	0.240
$\frac{3}{4}$	0.7500	10	0.334	0.302	16	0.373	0.351
$\frac{7}{8}$	0.8750	9	0.462	0.419	14	0.509	0.480
1	1.0000	8	0.606	0.551	12	0.663	0.625
$1\frac{1}{4}$	1.2500	7	0.969	0.890	12	1.073	1.024
$1\frac{1}{2}$	1.5000	6	1.405	1.294	12	1.581	1.521

*This table was compiled from ANSI B1.1-1974. The minor diameter was found from the equation $d_r = d - 1.299\ 038p$, and the pitch diameter from $d_m = d - 0.649\ 519p$. The mean of the pitch diameter and the minor diameter was used to compute the tensile-stress area.

Figure 8-3

Square thread; (b) Acme

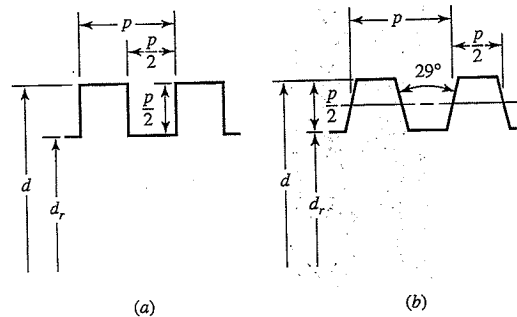


Table 8-3

Preferred Pitches for
Acme Threads

d , in	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3
p , in	$\frac{1}{16}$	$\frac{1}{14}$	$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

8-2 The Mechanics of Power Screws

A power screw is a device used in machinery to change angular motion into linear motion and, usually, to transmit power. Familiar applications include the lead screws of lathes and the screws for vises, presses, and jacks.

An application of power screws to a power-driven jack is shown in Fig. 8-4. You should be able to identify the worm, the worm gear, the screw, and the nut. Is the worm gear supported by one bearing or two?

In Fig. 8-5 a square-threaded power screw with single thread having a mean diameter d_m , a pitch p , a lead angle λ , and a helix angle ψ is loaded by the axial compressive force F . We wish to find an expression for the torque required to raise this load, and another expression for the torque required to lower the load.

Figure 8-4

The Joyce worm-gear screw jack. (Courtesy Joyce-Dayton Corp., Dayton, Ohio.)

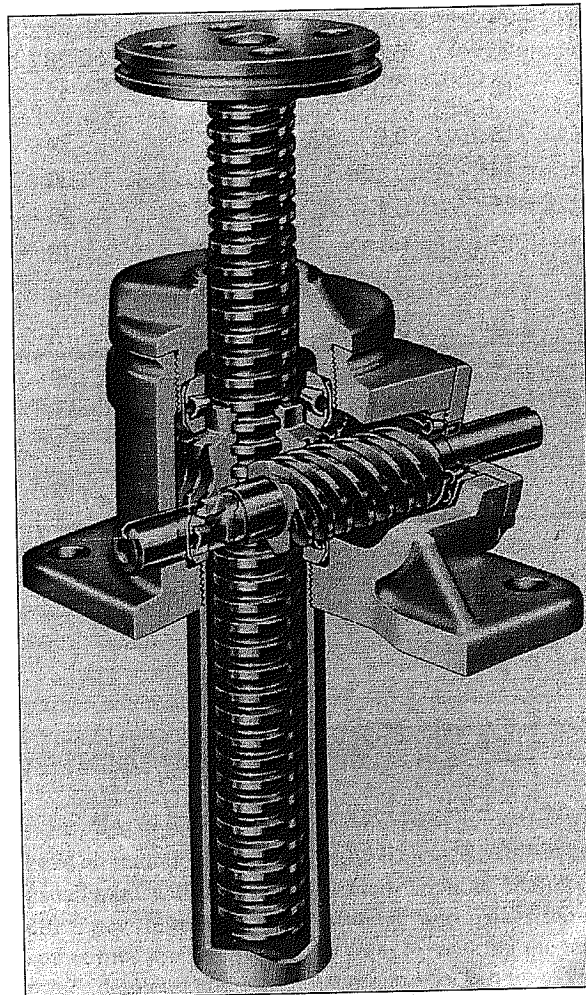
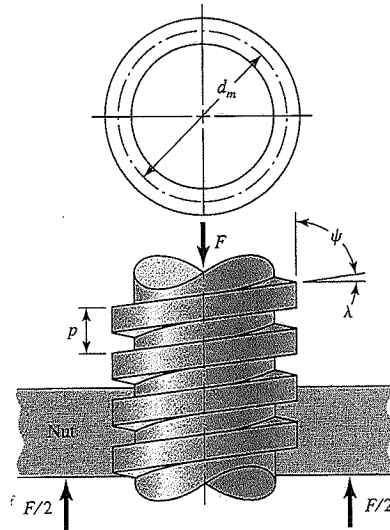


Figure 8-5

Diagram of a power screw.



First, imagine that a single thread of the screw is unrolled or developed (Fig. 8-6) for exactly the single turn. Then one edge of the thread will form the hypotenuse of a right triangle whose base is the circumference of the mean-thread-diameter circle and whose height is the lead. The angle λ , in Figs. 8-5 and 8-6, is the lead angle of the thread. We represent the summation of all the unit axial forces acting upon the normal thread area by F . To raise the load, a force P acts to the right (Fig. 8-6a), and to lower the load, P acts to the left (Fig. 8-6b). The friction force is the product of the coefficient of friction f with the normal force N , and acts to oppose the motion. The system is in equilibrium under the action of these forces, and hence, for raising the load, we have

$$\begin{aligned}\sum F_H &= P - N \sin \lambda - f N \cos \lambda = 0 \\ \sum F_V &= F + f N \sin \lambda - N \cos \lambda = 0\end{aligned}\quad (a)$$

In a similar manner, for lowering the load, we have

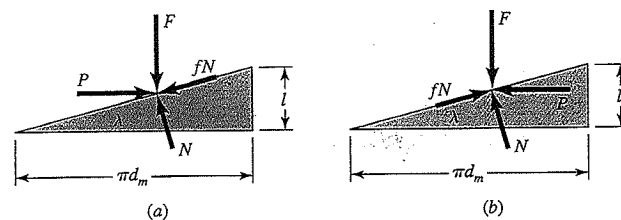
$$\begin{aligned}\sum F_H &= -P - N \sin \lambda + f N \cos \lambda = 0 \\ \sum F_V &= F - f N \sin \lambda - N \cos \lambda = 0\end{aligned}\quad (b)$$

Since we are not interested in the normal force N , we eliminate it from each of these sets of equations and solve the result for P . For raising the load, this gives

$$P = \frac{F(\sin \lambda + f \cos \lambda)}{\cos \lambda - f \sin \lambda}\quad (c)$$

Figure 8-6

Force diagrams: (a) lifting the load; (b) lowering the load.



and for lowering the load,

$$P = \frac{F(f \cos \lambda - \sin \lambda)}{\cos \lambda + f \cos \lambda} \quad (d)$$

Next, divide the numerator and the denominator of these equations by $\cos \lambda$ and use the relation $\tan \lambda = l/\pi d_m$ (Fig. 8-6). We then have, respectively,

$$P = \frac{F[(l/\pi d_m) + f]}{1 - (fl/\pi d_m)} \quad (e)$$

$$P = \frac{F[f - (l/\pi d_m)]}{1 + (fl/\pi d_m)} \quad (f)$$

Finally, noting that the torque is the product of the force P and the mean radius $d_m/2$ for raising the load we can write

$$T = \frac{F d_m}{2} \left(\frac{l + \pi f d_m}{\pi d_m - fl} \right) \quad (8-1)$$

where T is the torque required for two purposes: to overcome thread friction and to raise the load.

The torque required to lower the load, from Eq. (f), is found to be

$$T = \frac{F d_m}{2} \left(\frac{\pi f d_m - l}{\pi d_m + fl} \right) \quad (8-2)$$

This is the torque required to overcome a part of the friction in lowering the load. It may turn out, in specific instances where the lead is large or the friction is low, that the load will lower itself by causing the screw to spin without any external effort. In such cases, the torque T from Eq. (8-2) will be negative or zero. When a positive torque is obtained from this equation, the screw is said to be *self-locking*. Thus the condition for self-locking is

$$\pi f d_m > l$$

Now divide both sides of this inequality by πd_m . Recognizing that $l/\pi d_m = \tan \lambda$, we get

$$f > \tan \lambda \quad (8-3)$$

This relation states that self-locking is obtained whenever the coefficient of thread friction is equal to or greater than the tangent of the thread lead angle.

An expression for efficiency is also useful in the evaluation of power screws. If we let $f = 0$ in Eq. (8-1), we obtain

$$T_0 = \frac{Fl}{2\pi} \quad (8-4)$$

which, since thread friction has been eliminated, is the torque required only to raise the load. The efficiency is therefore

$$e = \frac{T_0}{T} = \frac{Fl}{2\pi T} \quad (8-5)$$

The preceding equations have been developed for square threads where the normal thread loads are parallel to the axis of the screw. In the case of Acme or other threads, the normal thread load is inclined to the axis because of the thread angle 2α and the lead angle λ . Since lead angles are small, this inclination can be neglected and only the effect of the thread angle (Fig. 8-7a) considered. The effect of the angle α is to increase the frictional force by the wedging action of the threads. Therefore the frictional terms in Eq. (8-1) must be divided by $\cos \alpha$. For raising the load, or for tightening a screw or bolt, this yields

$$T = \frac{Fd_m}{2} \left(\frac{l + \pi f d_m \sec \alpha}{\pi d_m - f l \sec \alpha} \right) \quad (8-5)$$

In using Eq. (8-5), remember that it is an approximation because the effect of the lead angle has been neglected.

For power screws, the Acme thread is not as efficient as the square thread, because of the additional friction due to the wedging action, but it is often preferred because it is easier to machine and permits the use of a split nut, which can be adjusted to take up for wear.

Usually a third component of torque must be applied in power-screw applications. When the screw is loaded axially, a thrust or collar bearing must be employed between the rotating and stationary members in order to carry the axial component. Figure 8-7b shows a typical thrust collar in which the load is assumed to be concentrated at the mean collar diameter d_c . If f_c is the coefficient of collar friction, the torque required is

$$T_c = \frac{F f_c d_c}{2} \quad (8-6)$$

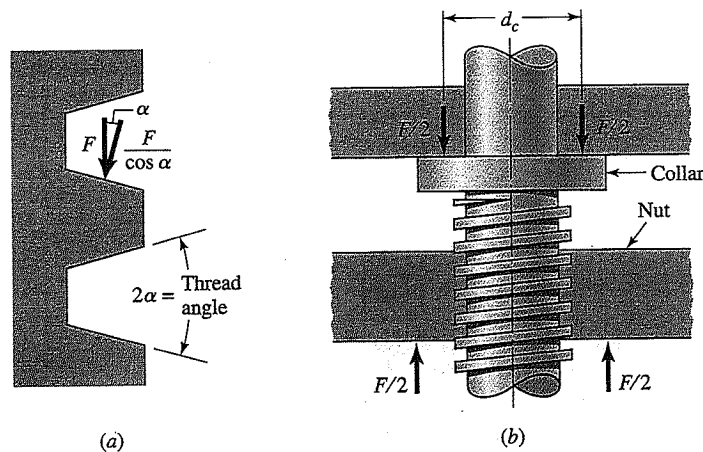
For large collars, the torque should probably be computed in a manner similar to that employed for disk clutches.

Nominal body stresses in power screws can be related to thread parameters as follows. The nominal shear stress τ in torsion of the screw body can be expressed as

$$\tau = \frac{16T}{\pi d_r^3} \quad (8-7)$$

Figure 8-7

(a) Normal thread force is increased because of angle α ;
 (b) thrust collar has frictional diameter d_c .



The axial stress σ in the body of the screw due to load F is

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d^2} \tag{8-8}$$

in the absence of column action. For a short column the J. B. Johnson equation is

$$\left(\frac{F}{A}\right)_{\text{crit}} = S_y - \left(\frac{S_y \ell}{2\pi k}\right) \frac{1}{CE} \tag{8-9}$$

after Eq. (4-53).

Nominal thread stresses in power screws can be related to thread parameters as follows. The bearing stress in Fig. 8-8, σ_B , is

$$\sigma_B = \frac{F}{\pi d_m n_t p/2} = \frac{2F}{\pi d_m n_t p} \tag{8-10}$$

where n_t is the number of engaged threads. The bending stress at the root of the thread σ_b is found from

$$\frac{I}{c} = \frac{(\pi d_r n_t) (p/2)^2}{6} = \frac{\pi}{24} d_r n_t p^2 \quad M = \frac{Fp}{4}$$

so

$$\sigma_b = \frac{M}{I/c} = \frac{Fp}{4} \frac{24}{\pi d_r n_t p^2} = \frac{6F}{\pi d_r n_t p} \tag{8-11}$$

The transverse shear stress τ at the center of the root of the thread due to load F is

$$\tau = \frac{3V}{2A} = \frac{3}{2} \frac{F}{\pi d_r n_t p/2} = \frac{3F}{\pi d_r n_t p} \tag{8-12}$$

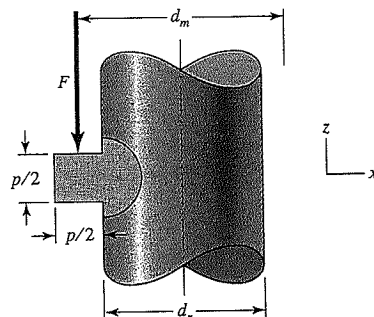
and at the top of the root it is zero. The von Mises stress σ' at the top of the root "plane" is found by first identifying the orthogonal normal stresses and the shear stresses. From the coordinate system of Fig. 8-8, we note

$$\begin{aligned} \sigma_x &= \frac{6F}{\pi d_r n_t p} & \tau_{xy} &= \frac{16T}{\pi d_r^3} \\ \sigma_y &= 0 & \tau_{yz} &= 0 \\ \sigma_z &= -\frac{4F}{\pi d_r^2} & \tau_{zx} &= 0 \end{aligned}$$

then use Eq. (h) of Sec. 6-7. Alternatively, one can recognize that this is a plane-stress situation, find the nonzero principal stresses, and find the von Mises stress using Eq. (6-13).

Figure 8-8

Geometry of square thread useful in finding bending and transverse shear stresses at the thread root.



The screw-thread form is complicated from an analysis viewpoint. Remember the origin of the tensile-stress area A_t , which comes from experiment. A power screw lifting a load is in compression and its thread pitch is *shortened* by elastic deformation. Its engaging nut is in tension and its thread pitch is *lengthened*. The engaged threads cannot share the load equally. Some experiments show that the first engaged thread carries 0.38 of the load, the second 0.25, the third 0.18, and the seventh is free of load. In estimating thread stresses using the equations above, substituting $0.38F$ for F and setting n_t to 1 will give the largest level of stresses in the thread-nut combination.

EXAMPLE 8-1

A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with double threads, and it is to be used in an application similar to Fig. 8-4. The given data include $f = f_c = 0.08$, $d_c = 40$ mm, and $F = 6.4$ kN per screw.

- Find the thread depth, thread width, pitch diameter, minor diameter, and lead.
- Find the torque required to raise and lower the load.
- Find the efficiency during lifting the load.
- Find the body stresses, torsional and compressive.
- Find the bearing stress.
- Find the thread stresses bending at the root, shear at the root, and von Mises stress and maximum shear stress at the same location.

Solution From Fig. 8-3a the thread depth and width are the same and equal to half the pitch, or 2 mm. Also

$$d_m = d - p/2 = 32 - 4/2 = 30 \text{ mm}$$

Answer $d_r = d - p = 32 - 4 = 28$ mm

$$l = np = 2(4) = 8 \text{ mm}$$

(b) Using Eqs. (8-1) and (8-6), and assuming positive torque is a load-raising torque, the torque required to turn the screw against the load is

$$T = \frac{Fd_m}{2} \left(\frac{l + \pi f d_m}{\pi d_m - fl} \right) + \frac{Ff_c d_c}{2}$$

$$= \frac{6.4(30)}{2} \left[\frac{8 + (0.08)(30)}{(30) - 0.08(8)} \right] + \frac{6.4(0.08)40}{2}$$

Answer $= 15.94 + 10.24 = 26.18 \text{ N} \cdot \text{m}$

Using Eqs. (8-2) and (8-6), negative torque being a load-lowering torque,

$$T = \frac{-Fd_m}{2} \left(\frac{\pi f d_m - l}{\pi d_m + fl} \right) - \frac{Ff_c d_c}{2}$$

$$= \frac{-6.4(30)}{2} \left[\frac{\pi(0.08)30 - 8}{\pi(30) + 0.08(8)} \right] - \frac{6.4(0.08)(40)}{2}$$

Answer $= +0.466 - 10.24 = -9.77 \text{ N} \cdot \text{m}$

The plus sign in the first term indicates that the screw alone is not self-locking and would rotate under the action of the load except for the fact that the collar friction is present and must be overcome, too. Thus the torque required to rotate the screw "with" the load is less than is necessary to overcome collar friction alone.

(c) The overall efficiency in raising the load is

$$\text{Answer } e = \frac{Fl}{2\pi T} = \frac{6.4(8)}{2\pi(26.18)} = 0.311$$

(d) The body shear stress τ due to torsional moment T at the outside of the screw body is

$$\text{Answer } \tau = \frac{16T}{\pi d_r^3} = \frac{16(26.18)(10^3)}{\pi(28^3)} = 6.07 \text{ MPa}$$

The axial nominal normal stress σ is

$$\text{Answer } \sigma = \frac{4F}{\pi d_r^2} = \frac{4(6.4)10^3}{\pi(28^2)} = -10.39 \text{ MPa}$$

(e) The bearing stress σ_B is, with one thread carrying $0.38F$,

$$\text{Answer } \sigma_B = \frac{2(0.38F)}{\pi d_r n_r p} = \frac{2(0.38)(6.4)10^3}{\pi(30)(1)(4)} = 12.9 \text{ MPa (compressive)}$$

(f) The thread-root bending stress σ_b with one thread carrying $0.38F$ is

$$\sigma_b = \frac{6(0.38F)}{\pi d_r n_r p} = \frac{6(0.38)(6.4)10^3}{(28)(1)(4)} = 41.5 \text{ MPa}$$

The transverse shear at the extreme of the root cross section due to bending is zero. However, there is a circumferential shear stress at the extreme of the root cross section of the thread as shown in part (d) of 6.07 MPa. The three-dimensional stresses, after Fig. 8-8, noting the y coordinate is into the page, are

$$\begin{aligned} \sigma_x &= 41.5 \text{ MPa} & \tau_{xy} &= 6.07 \text{ MPa} \\ \sigma_y &= 0 & \tau_{yz} &= 0 \\ \sigma_z &= -10.39 \text{ MPa} & \tau_{zx} &= 0 \end{aligned}$$

Equation (h) of Sec. 6-7 can be written as

$$\begin{aligned} \sigma' &= \frac{1}{\sqrt{2}} \left\{ (41.5 - 0)^2 + [0 - (-10.39)]^2 + (-10.39 - 41.5)^2 + 6(6.07)^2 \right\}^{1/2} \\ &= 48.7 \text{ MPa} \end{aligned}$$

Alternatively, this can be viewed as a plane-stress situation with

$$\begin{aligned} \sigma_x &= 41.5 \text{ MPa} & \tau_{xy} &= 6.07 \text{ MPa} \\ \sigma_y &= -10.39 \text{ MPa} & \tau_{yx} &= 0 \end{aligned}$$

The principal stresses, nonzero, are

$$\begin{aligned} \sigma_1, \sigma_2 &= \frac{41.5 + (-10.39)}{2} \pm \sqrt{\left[\frac{41.5 - (-10.39)}{2} \right]^2 + 6.07^2} \\ &= 15.56 \pm 26.65 = 42.21, -11.09 \text{ MPa} \end{aligned}$$

From Eq. (6-13),

$$\text{Answer } \sigma' = \left[42.21^2 - 42.21(-11.09) + (-11.09)^2 \right]^{1/2} = 48.7 \text{ MPa}$$

Since we have the principal stresses, the maximum shear stress can be found by ordering the principal stresses as 42.21, 0, -11.09 MPa and following Eqs. (3-11), which amount to

$$\begin{aligned} \text{Answer } \tau_{\max} &= \max \left[\frac{42.21 - 0}{2}, \frac{0 - (-11.09)}{2}, \frac{42.21 - (-11.09)}{2} \right] \\ &= \max[21.11, 5.55, 26.65] = 26.65 \text{ MPa} \end{aligned}$$