Mechatronics Design – Class#15

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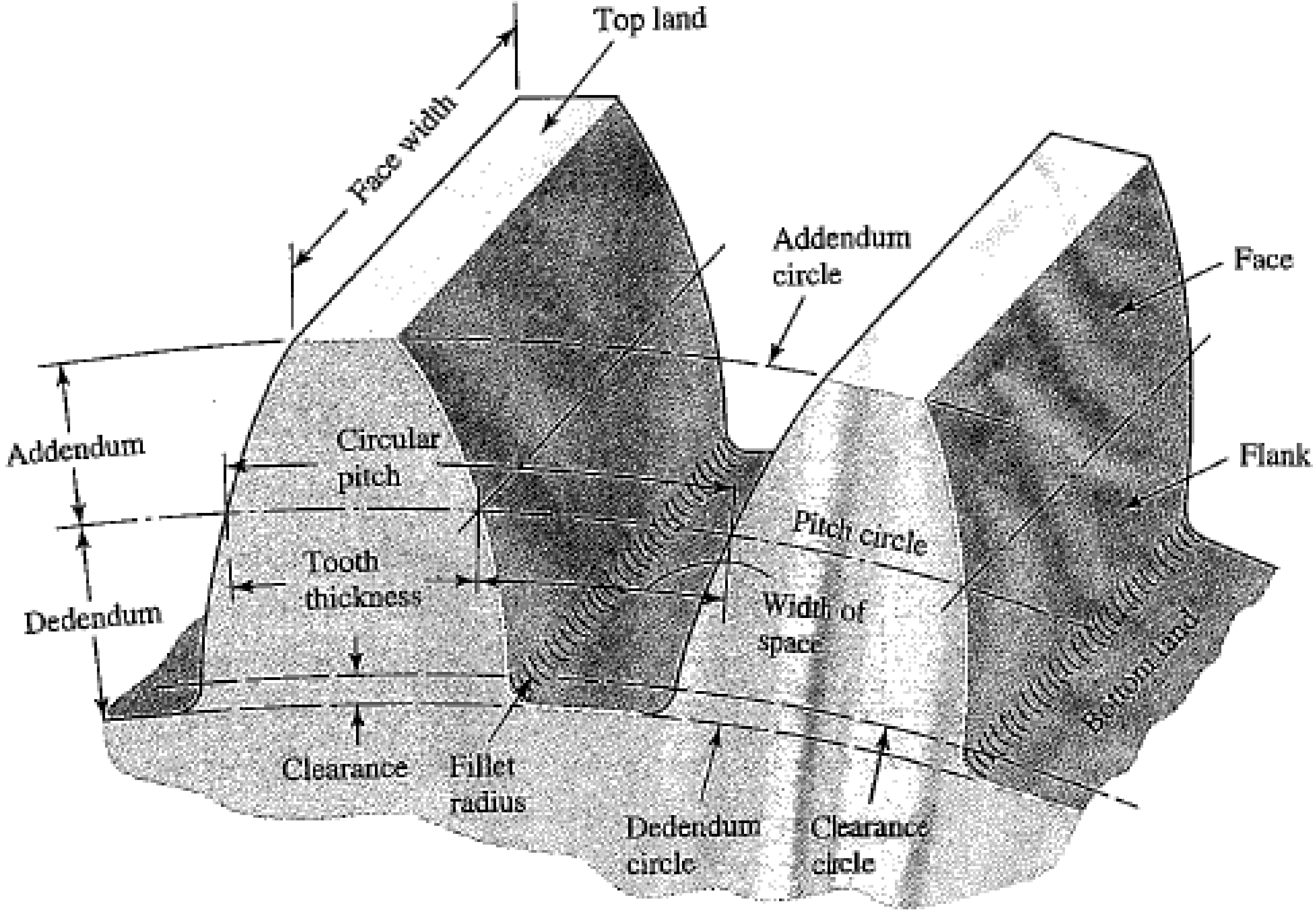
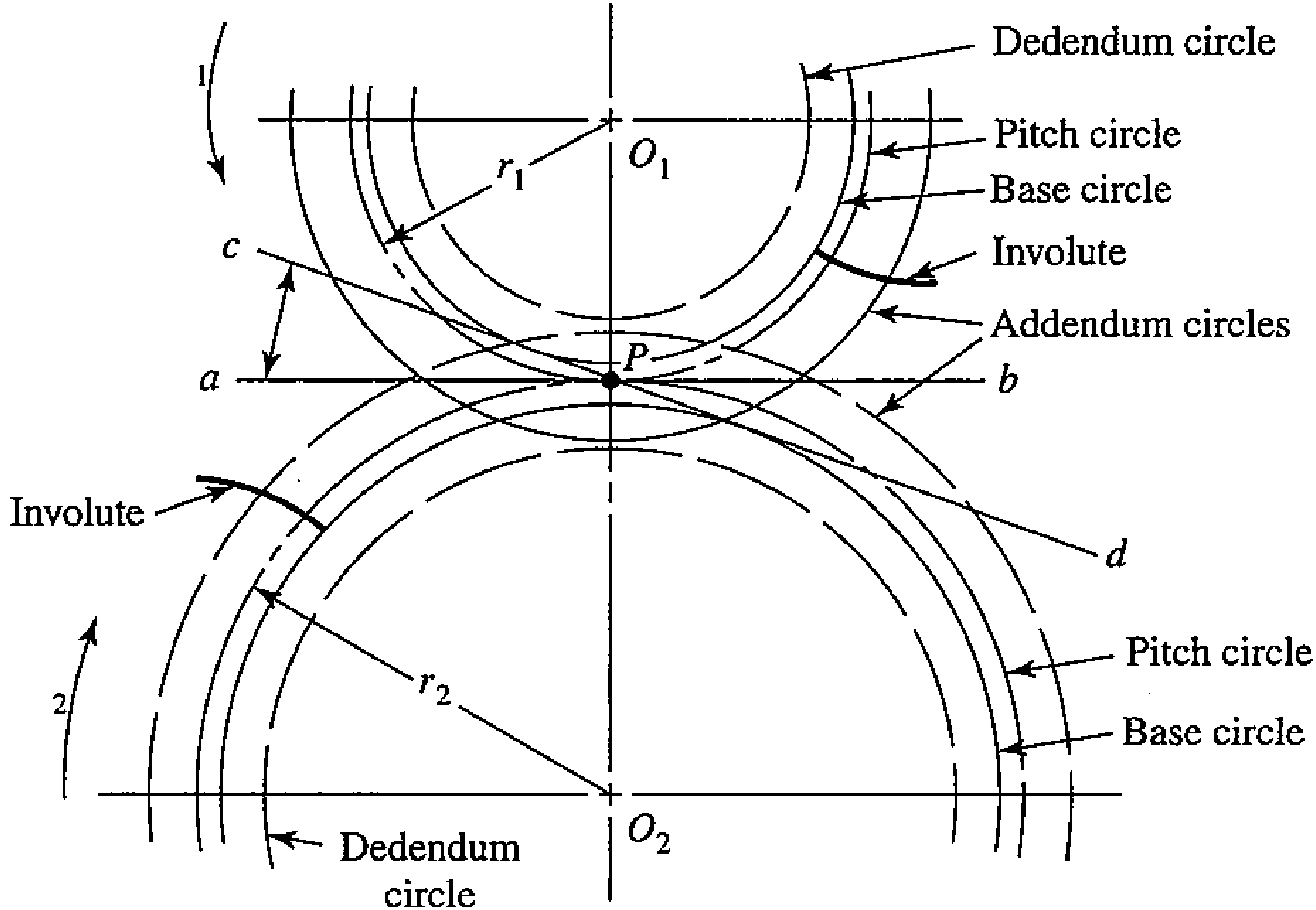
Outline

- Review
  - Gears fundamentals
- Involute
- Contact length & Contact Ratio
- HW#3 (due next Monday)

Gears


Top land
Face width
Addendum circle
Face
Flank
Addendum
Circular pitch
Pitch circle
Tooth thickness
Dedendum
Width of space
Bottom land
Clearance
Fillet radius
Dedendum circle
Clearance circle

Gear Circles



Basic Numbers

d = pitch diameter

m = module $= \frac{d_2}{N_2} = \frac{d_1}{N_1}$

p = circular pitch $= \frac{\pi d}{N}$

P = diametral pitch $= \frac{N}{d}$

$a = \frac{1}{P}$ — standard

$b = \frac{1.25}{P}$ — standard

Conjugate Action

When tooth profiles are designed to produce a constant angular velocity ratio during meshing

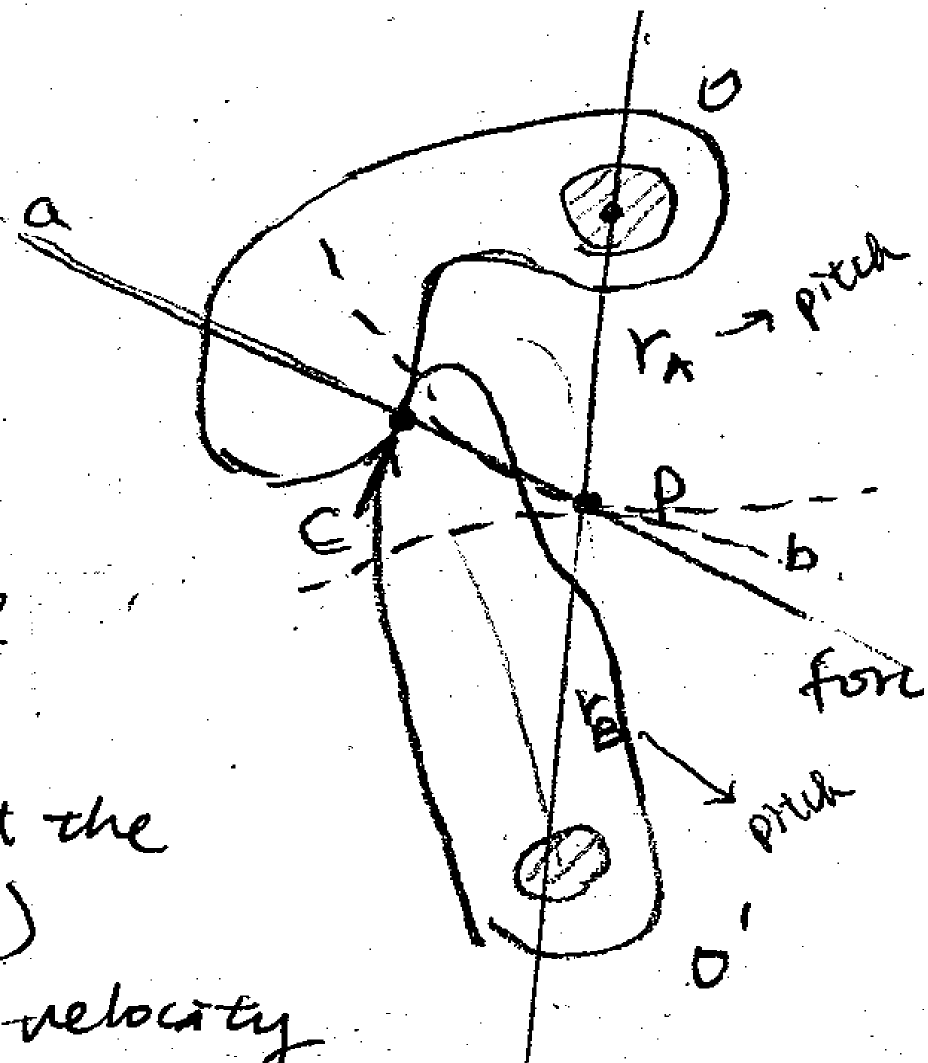
In theory, given one tooth profile, one can fine the profile for meshing tooth for conjugate action

One solution is "involute profile" ⟶ universal use for gears

Graphic Explanation

C is the contact point
where two surfaces are
tangent to each other

if P is not changing,
O & O' do not feel
the change
(no matter what the
shapes are)
⇒ constant angular velocity



Line of Action

force is acting on common normal $\overline{ab}$.

$\boxed{\text{line of action} \atop \text{pressure line}}$

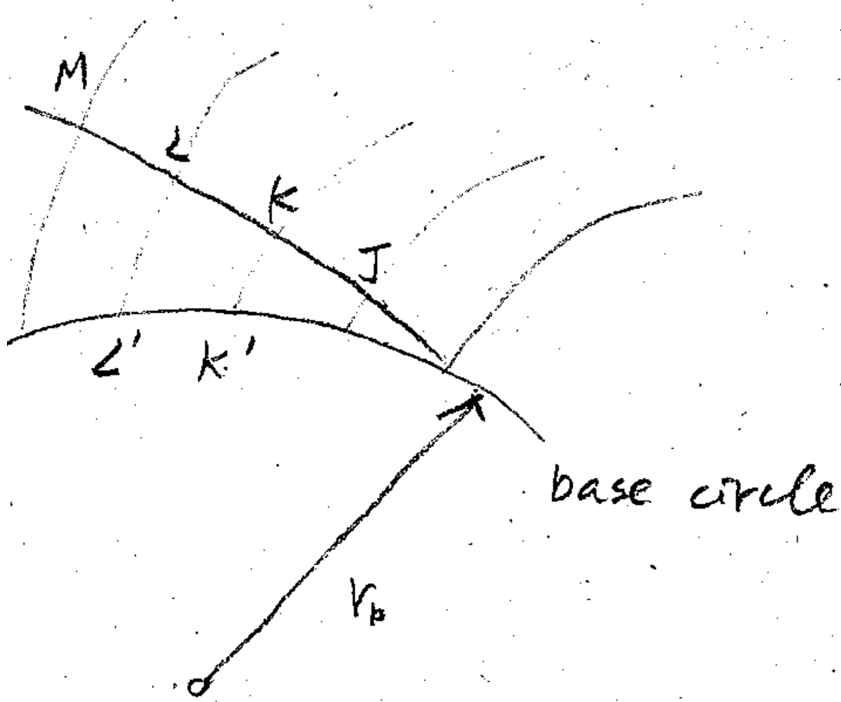
P is the intersection point

of $\overline{ab}$ & $\overline{OO'}$

$\Rightarrow$ pitch point

Involute

locus traced by the free end of a taut string that is "unwrapped" from the circle

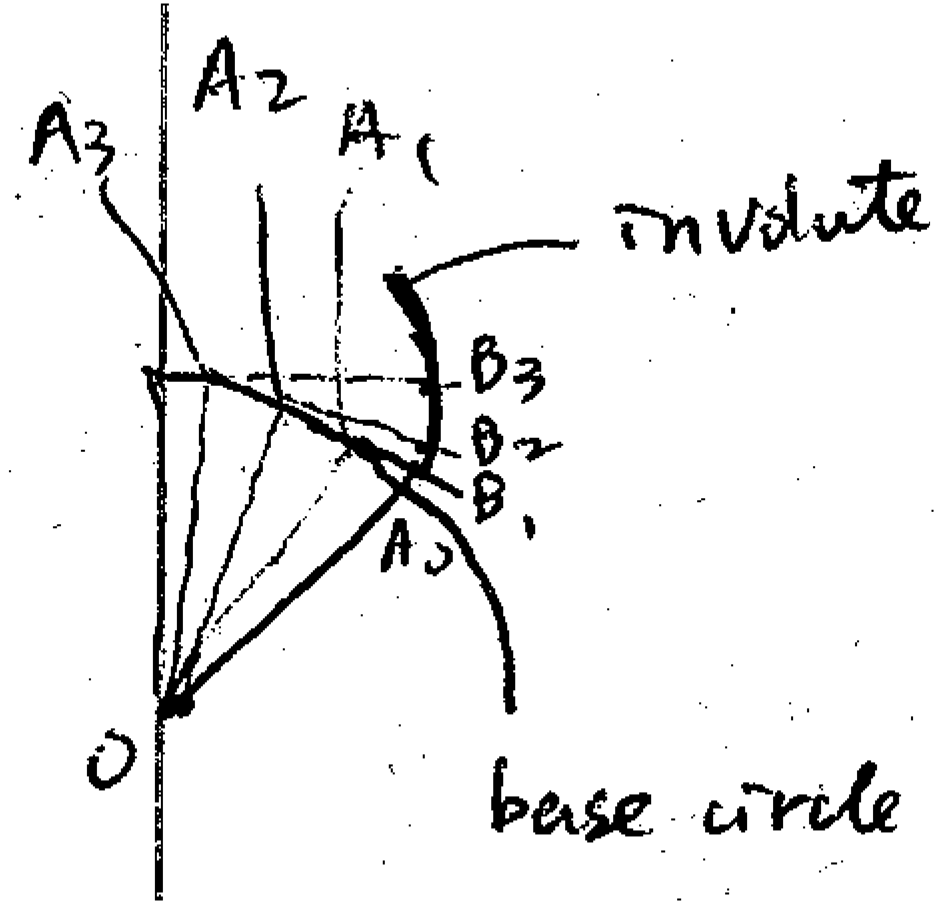


$$\overline{ML} = \overline{LK} = \overline{KJ} = \cdots$$

taut string property

$$\overline{LK} = \widehat{L'K'} = \frac{2\pi r_b}{N}$$

Making Involute Profile



$\overline{A_0A_1} = \overline{A_1A_2} = \overline{A_2A_3} = \cdots$

make tangent lines on

$A_1, A_2, A_3 \cdots$

along A_1 tangent line

find B_1 such that

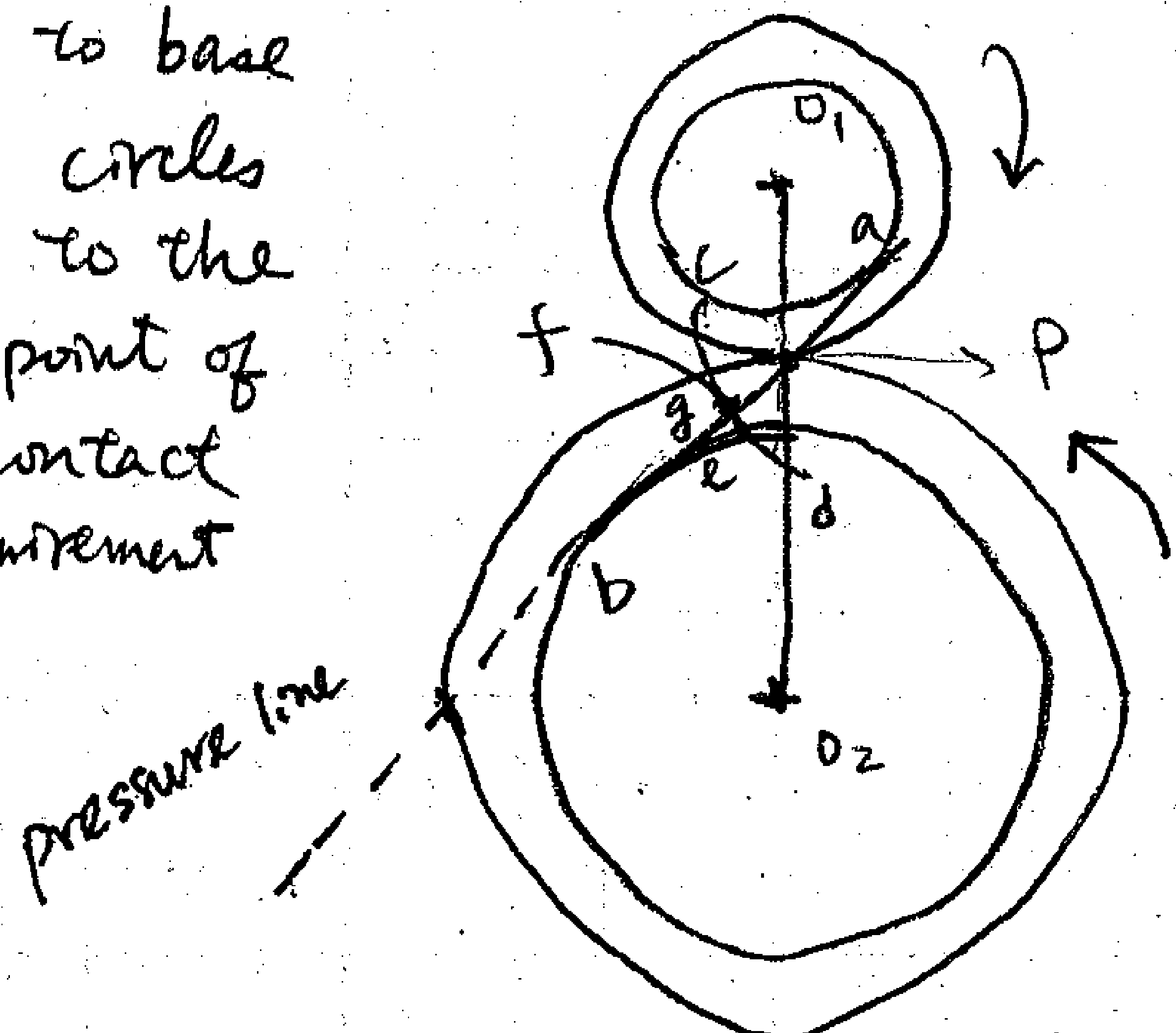
$\overline{A_1B_1} = \overline{A_1A_0}$

Pressure Line

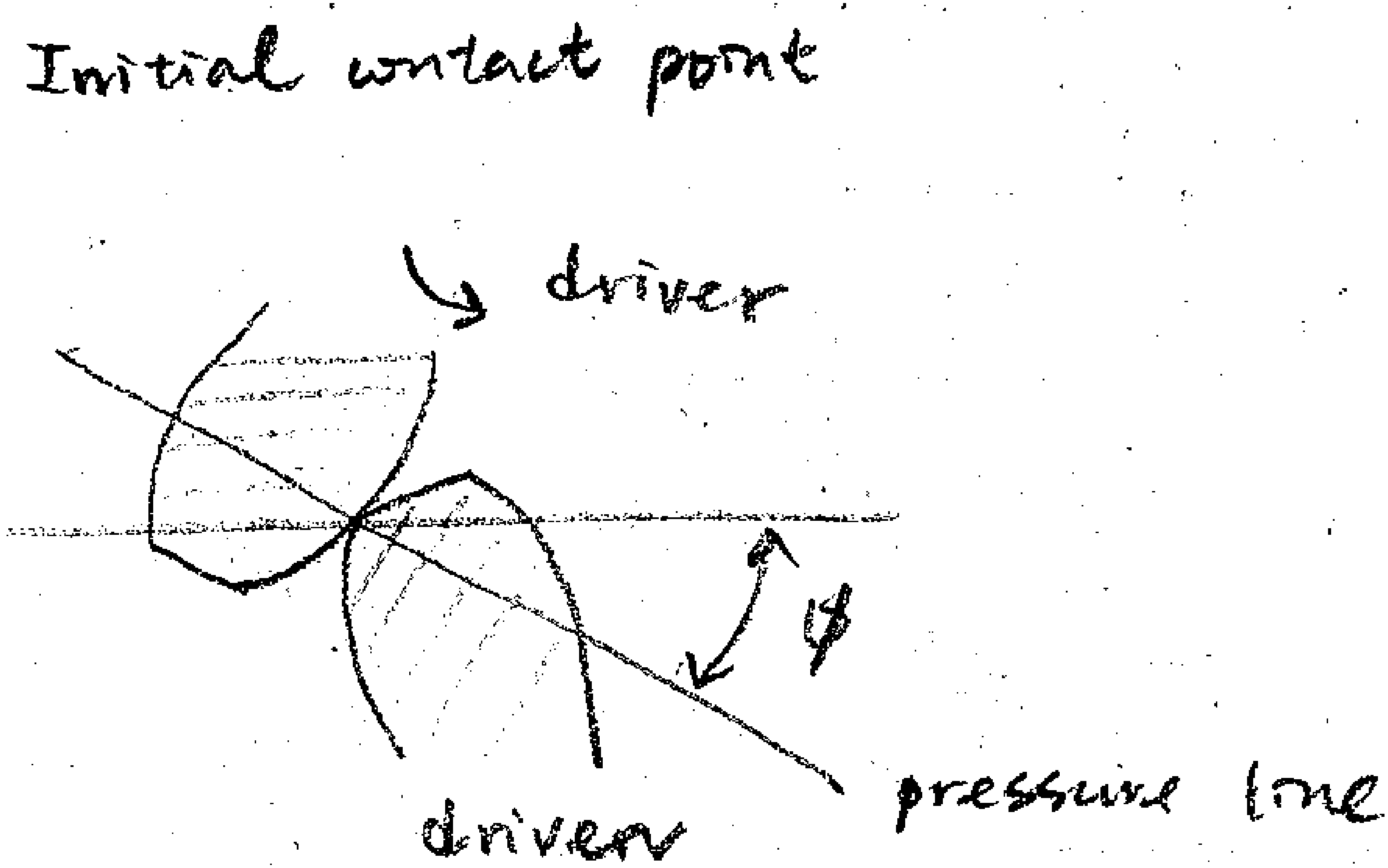
→ always tangent to base circles

→ - - - normal to the involute at point of contact

⇒ satisfy the requirement

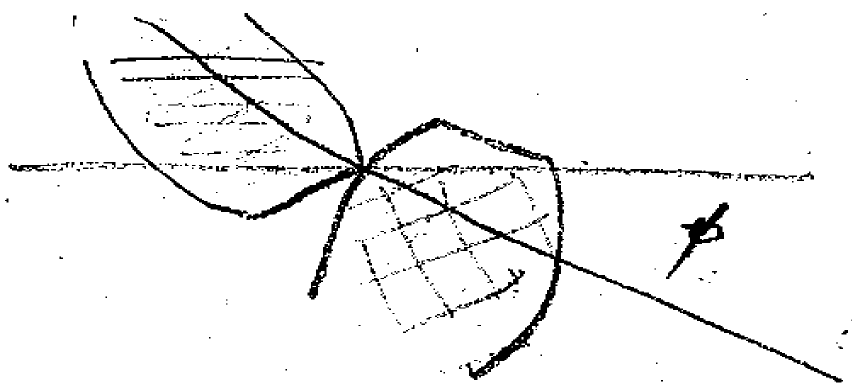


Initial Contact



Final Contact

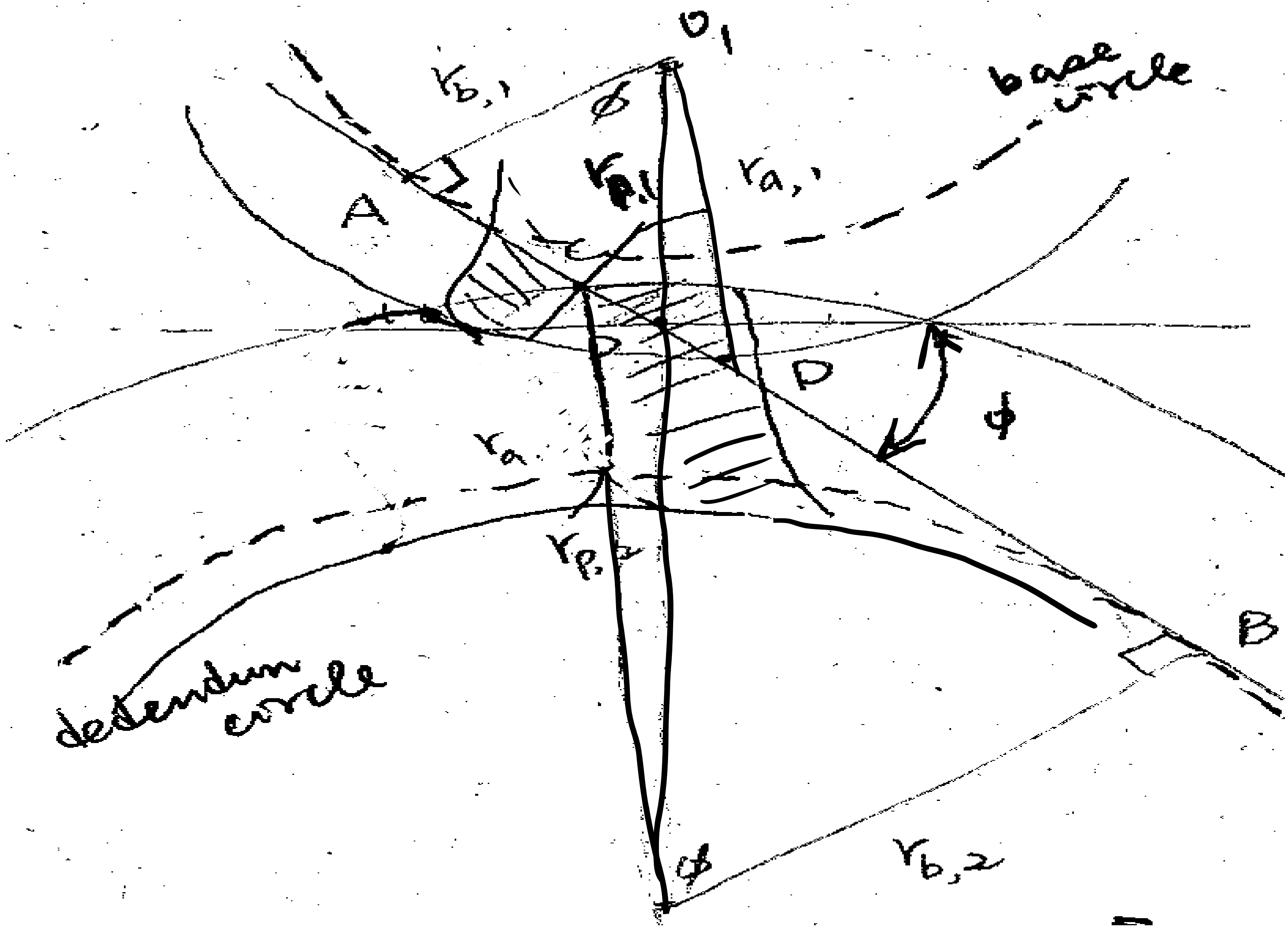
final contact



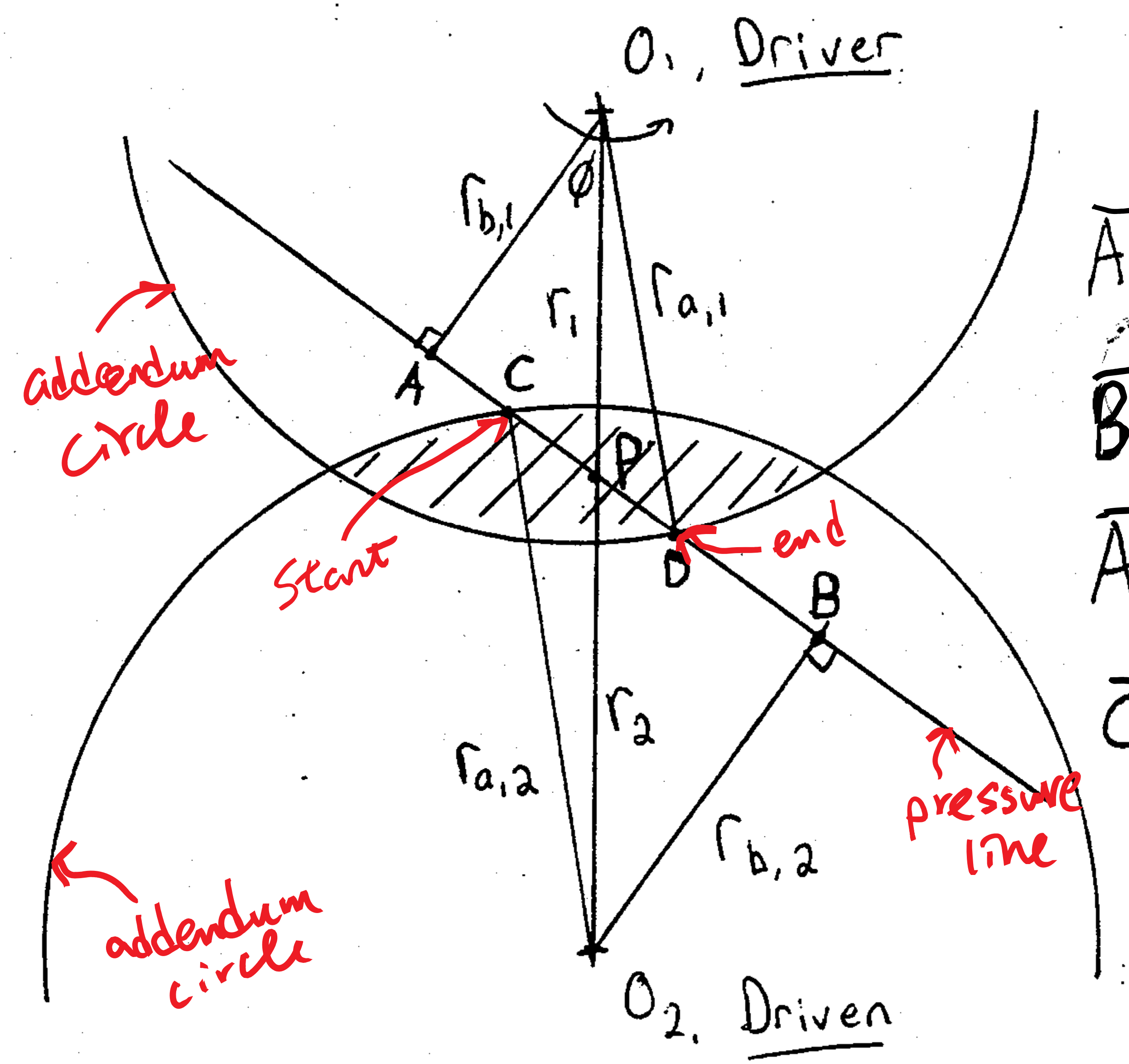
contact starts when addendum circle of driven gear intersects the pressure line

ends when addendum circle of driver gear intersects the pressure line

Contact - Graphics



More Clean Figure



$$\overline{AD} = \sqrt{r_{a,1}^2 - r_{b,1}^2}$$

$$\overline{BC} = \sqrt{r_{a,2}^2 - r_{b,2}^2}$$

$$\overline{AB} = O_1O_2 \sin\phi$$

$$\overline{CD} = \overline{AD} + \overline{BC} - \overline{AB}$$

Contact Ratio

- A + B are pts of tangency to base circles
- r_1 + r_2 are radii of pitch circles
- Remember line $\overline{AB}$ is fixed throughout contact
- Circles drawn here are addendum circles

$$C.R. = \frac{CD}{2\pi r_b / N}$$

where $\frac{r_b}{N} = \frac{r_{b,1}}{N_1} = \frac{r_{b,2}}{N_2}$

In general:

$$1.5 < C.R. < 2.0$$

Detailed Numbers

$$\overline{AD} = \sqrt{r_{a1}^2 - r_{b1}^2}$$

$$\overline{BC} = \sqrt{r_{a2}^2 - r_{b2}^2}$$

$$\left\{ \begin{array}{l} r_b = r_p \cos\phi \\ r_a = r_p + \frac{1}{P} \end{array} \right.$$

$$\overline{AB} = r_{p1} \sin\phi + r_{p2} \sin\phi = O_1 O_2 \sin\phi$$

$$\overline{CD} = \overline{AD} + \overline{BC} - \overline{AB}$$

Interference

$BC > AB$

or

$AD > AB$

Example

1) Given: $N_p = 24T$, $N_g = 48T$, $P = 4\frac{T}{in}$, $\phi = 25^\circ$

$$r_p = \frac{1}{2}\frac{N_p}{P} = \frac{1}{2}\frac{24}{4} = 3\,in, \quad r_g = \frac{1}{2}\cdot\frac{48}{4} = 6\,in$$

$$r_{a,p} = r_p + a = r_p + \frac{1}{P} = 3 + \frac{1}{4} = 3.25\,in, \quad r_{a,g} = 6 + \frac{1}{4} = 6.25\,in$$

$r_{b,p} = r_p \cos\phi = 3\cos 25^\circ = 2.72\,in \qquad r_{b,g} = 6\cos 25^\circ = 5.44\,in$

$$AD = \sqrt{r_{a,p}^2 - r_{b,p}^2} = \sqrt{3.25^2 - 2.72^2} = 1.78\,in$$

$$BC = \sqrt{r_{a,g}^2 - r_{b,g}^2} = \sqrt{6.25^2 - 5.44^2} = 3.08\,in$$

$$AB = (r_p + r_g)\sin\phi = (3+6)\sin 25^\circ = 3.80\,in$$

$$C.R. = \frac{AD+BC-AB}{2\pi r_b/N} = \frac{1.78+3.08-3.80}{2\pi\,(2.72/24)} = \boxed{1.49\ \text{teeth}}$$