3. planetary gear trains ("imagining that arm is fixed")

most general formula

\[ \frac{W_p - W_h}{W_k - W_h} \rightarrow \text{you are sitting on the arm and observing the moves of others} \]

- \( W_p \): first gear angular velocity
- \( W_h \): arm " ~ "
- \( W_k \): last " ~ "

ring

planet

arm

Sun

(actual planetary gear trains involve two or more equally spaced planets to balance forces)

EX: no ring \( \rightarrow \) external mesh

\[ \begin{align*}
& \text{if a known speed is applied to arm, what is the absolute rotation of the planet?} \\
& W_p = W_a + W_{p/a} \quad (W_p - W_h) \\
& \Rightarrow \frac{W_p}{W_h} = 1 + \frac{W_{p/a}}{W_a} \\
& \text{sun is fixed} \Rightarrow W_a = W_b + W_{p/a} W_{p/b} \quad \text{planet speed w.r.t. arm} \\
& \Rightarrow \frac{W_p}{W_h} = 1 + \frac{W_{p/a}}{W_{p/b}} = 1 - \frac{W_{p/a}}{W_{p/b}} \\
& \Rightarrow \frac{W_{p/a}}{W_{p/b}} = - \frac{N_p}{N_a} \\
& \text{note: the sign changes} \Rightarrow W_p = W_a (1 + \frac{N_p}{N_a}) \quad \text{planet moves} (1 + \frac{N_p}{N_a}) \text{ rev} \\
& \text{when arm moves} \}
\]
In general, if gear $i$ meshes with gear $j$ then:

$$ W_{iA} = \sin \theta \frac{W_{jA}}{N_j/N_i} $$

$$ \sin \theta = \begin{cases} 1 & \text{internal mesh} \\ -1 & \text{external mesh} \end{cases} $$

DX hand out

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{A}$</td>
<td>$W_{A}$</td>
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</tr>
<tr>
<td>$W_{iB}$</td>
<td>$-W_{A}$</td>
<td>$-W_{A}$</td>
<td>$-W_{A}$</td>
<td>$-W_{iA}$</td>
<td>$-W_{iA}$</td>
<td>$-W_{iA}$</td>
</tr>
<tr>
<td>$W_{iC}$</td>
<td>$-4W_{A}$</td>
<td>$-W_{A}$</td>
<td>$-W_{A}$</td>
<td>$-W_{iA}$</td>
<td>$-W_{iA}$</td>
<td>$-W_{iA}$</td>
</tr>
</tbody>
</table>

In internal meshing:

- $B \rightarrow$ grounded, $w_B = 0$
- $C \rightarrow B \& C$ meshing gears, $\frac{W_{C}}{W_{B}} = \frac{N_B}{N_C} = \frac{100}{50} = 2$ [Ans.]
- $D \rightarrow$ same as $C$ (same shaft)
- $E \rightarrow \frac{W_{E}}{W_{A}} = \frac{N_D}{N_E} = \frac{35}{105}$ $\Rightarrow W_{E} = \frac{1}{3} W_{A}$
- $F \rightarrow$ same as $E$

1. $W_F = -\frac{4}{3} W_A = -\frac{8000}{3}$ rpm
2. $F, G$ spur gears

$$ \frac{W_F}{N_F} = \frac{4}{21} W_A N_F = \frac{200}{21} \Rightarrow N_F = 200 $$

Now solve $W_A$:

- $W_F = W_A$ reverse
- $W_F = W_A$ reverse $\Rightarrow W_A = W_A$ same direction

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**Example:** Use 1.3.31 yourselves
Basic steps to solve gear train problems

1. Planetary
   - Gear trains must have two inputs

2. Derive relationship of these two inputs, and find \( \omega_a \)

3. \[
\frac{\omega_{\text{final}}}{\omega_{\text{initial}}} = \frac{\omega_f}{\omega_i} = \frac{N_g}{N_F} \]

Redo example

1. Inputs => \( \omega_A = \omega_B = 0 \), ok
2. \( \omega_a = \omega_B \), ok
3. \[
\frac{\omega_f}{\omega_a} = \frac{\omega_B}{\omega_A} \cdot \frac{N_B}{N_E} = \omega_a \cdot \frac{N_B}{N_E} \]

\[
\omega_f = \omega_a + (\omega_a - \omega_B) = \omega_a - \omega_a = \frac{25}{21}
\]

F & G are boxed regular gears

\[
\frac{N_g}{N_F} \quad \Rightarrow \quad N_g = 20
\]

\[
\frac{50}{41} = \frac{N_F}{N_F} \quad N_F = 200 \quad N_g = 200
\]

\[
\frac{50}{31} = \frac{N_g}{N_F} \quad N_F = 200 \quad N_g = 200
\]