③ planetary gear trains ("imagining that arm is fixed")

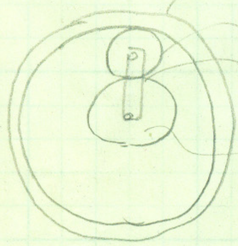
most general formula

train ratio $e = \dfrac{\omega_L - \omega_A}{\omega_F - \omega_A}$ → you are sitting on the arm and observing the moves of others

ω_F — first gear angular velocity

ω_A — arm " "

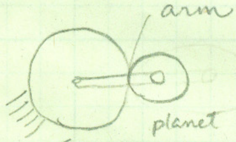
ω_L — last " " "



write it down at hand outs

(actual planetary gear trains involve two or more equally spaced planets to balance forces)

ex) no ring → external mesh



if a known speed ω_a is applied to arm, what is the absolute rotation of the planet?

✓ ① inputs ⇒ ω_a, $\omega_s = 0$

✓ ② $\omega_a = \omega_a$

③ $\dfrac{\omega_{p/a}}{\omega_{s/a}} = -\dfrac{N_s}{N_p}$

$\omega_p - \omega_a = (\overset{0}{\omega_s} - \omega_a)\left(-\dfrac{N_s}{N_p}\right)$

$\omega_p = \omega_a\left(1 + \dfrac{N_s}{N_p}\right)$

$\omega_p = \omega_a + \omega_{p/a}$ → $(\omega_p - \omega_a)$

planet speed w.r.t. arm

⇒ $\dfrac{\omega_p}{\omega_a} = 1 + \dfrac{\omega_{p/a}}{\omega_a}$

sun is fixed ⇒ $\omega_a = \omega_s + \omega_{a/s}$

⇒ $\dfrac{\omega_p}{\omega_a} = 1 + \dfrac{\omega_{p/a}}{\omega_{a/s}} = 1 - \underbrace{\boxed{\dfrac{\omega_{p/a}}{\omega_{s/a}}}}_{e}$

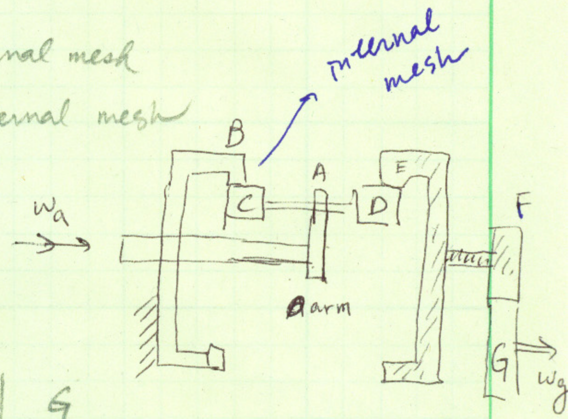
$\dfrac{\omega_{p/a}}{\omega_{s/a}} = -\dfrac{N_s}{N_p}$

note the sign changes ⇒ $\omega_p = \omega_a\left(1 + \dfrac{N_s}{N_p}\right)$ when arm moves 1 rev, planet moves $\left(1 + \dfrac{N_s}{N_p}\right)$ rev

☆ In general if gear i meshes with gear j

then $$w_{j/a} = \text{sing}\ w_{i/a}\frac{N_i}{N_j}$$

$$\text{sing} = \begin{cases} + & \text{internal mesh} \\ - & \text{external mesh}\end{cases}$$



ex) hand out

	A	B	C	D	E	F	G
w_{arm}	w_a	w_a	w_a	w_a	w_a	w_a	w_a
$w_i/_{arm}$	0	$-w_a$	$-5w_a$	$-5w_a$	$-\frac{25}{21}w_a$	$-\frac{25}{21}w_a$	$-\frac{N_F}{N_G}\cdot(-\frac{25}{21}w_a)$
$w_i = w_a + w_{i/arm}$	w_a	0	$-4w_a$	$-4w_a$	$-\frac{4}{21}w_a$	$-\frac{4}{21}w_a$	

Q: (spur gear ratio)

$B = 100\ T$
$C = 20\ T$
$D = 25\ T$
$E = 105\ T$
$w_a = 1000$ rpm
$w_g = 20$ rpm

B → grounded, $w_B = 0$

C → B & C meshing gears, $\frac{w_{C/a}}{w_{B/a}} = +\frac{N_B}{N_C} = \frac{100}{20} = 5$ (internal meshing)

$\Rightarrow w_{C/a} = +5 w_{B/a} = -5w_a$

D → same as C (same shaft)

E → $\frac{w_{E/a}}{w_{D/a}} = +\frac{N_D}{N_E} = \frac{25}{105}$, $\Rightarrow w_{E/a} = -5\cdot\frac{25}{105}w_a$

F → same as E

① $w_F = -\frac{4}{21}w_a = -\frac{4000}{21}$ rpm

② F, G spur gears

$$\frac{w_F}{w_G} = -\frac{N_G}{N_F}.\quad N_G = \frac{-\frac{4}{21}w_a N_F}{w_g} = \frac{4}{21}\frac{1000}{20}N_F = \frac{200}{21}N_F$$

choose $N_F = 21 \Rightarrow N_g = 200$

③ w_F & w_a reverse
w_F & w_G reverse $\Rightarrow$ $\boxed{w_a \text{ \& } w_G \text{ same direction}}$

④ ~~read example 13.3, 13.4 yourself~~

- Basic steps to solve planetary gear train problems

① planetary gear trains must have two inputs

② derive relationship of these two inputs, and find ω_a

③ " " " $\frac{\omega_{Final/a}}{\omega_{Initial/a}}$, find $\omega_{Final/a}$

☆ redo example

① inputs ⇒ ω_a
$\omega_B = 0$, ok

② $\omega_a = \omega_a$, ok

③ $\omega_{F/a} \overset{=\omega_{E/a}}{=} \omega_{D/a} \cdot \frac{N_D}{N_E}$

$= \omega_{B/a} \cdot \frac{N_B}{N_G} \cdot \frac{N_D}{N_E}$

$\omega_F = \omega_a + (0 - \omega_a) \cdot \frac{100}{20} \cdot \frac{25}{105}$

$= \omega_a - \omega_a \cdot \frac{25}{21}$

$= -\frac{4}{21} \omega_a$ ※

F & G are ~~fixed~~ regular gears

$\frac{\omega_F}{\omega_G} = \frac{N_G}{N_F} \Rightarrow \ldots$ ~~$N_F = 20$~~

$\frac{-\frac{4}{21} \cdot \cancel{1000}^{50}}{\cancel{20}} = \frac{N_G}{N_F}$ $N_G = 200$
$N_F = 21$