

# Chapter 8

## DEFLECTION OF BEAMS BY INTEGRATION

### 8.1. INTRODUCTION

We saw in Sec. 4.4 that a prismatic beam subjected to pure bending is bent into an arc of circle and that, within the elastic range, the curvature of the neutral surface may be expressed as

$$\frac{1}{\rho} = \frac{M}{EI} \quad (4.21)$$

where  $M$  is the bending moment,  $E$  the modulus of elasticity, and  $I$  the moment of inertia of the cross section about its neutral axis.

When a beam is subjected to a transverse loading, Eq. (4.21) remains valid for any given transverse section, provided that Saint-Venant's principle applies. However, both the bending moment and the curvature of the neutral surface will vary from section to section. Denoting by  $x$  the distance of the section from the left end of the beam, we write

$$\frac{1}{\rho} = \frac{M(x)}{EI} \quad (8.1)$$

Consider, for example, a cantilever beam  $AB$  of length  $L$  subjected to a concentrated load  $P$  at its free end  $A$  (Fig. 8.1a). We have  $M(x) = -Px$  and, substituting into (8.1),

$$\frac{1}{\rho} = -\frac{Px}{EI}$$

which shows that the curvature of the neutral surface varies linearly with  $x$ , from zero at  $A$ , where  $\rho_A$  itself is infinite, to  $-PL/EI$  at  $B$ , where  $|\rho_B| = EI/PL$  (Fig. 8.1b).

Consider now the overhanging beam  $AD$  of Fig. 8.2, which supports two concentrated loads as shown. From the free-body diagram of the beam (Fig. 8.3a), we find that the reactions at the supports are  $R_A = 1$  kN and  $R_C = 5$  kN, respectively, and draw the corresponding bending-moment

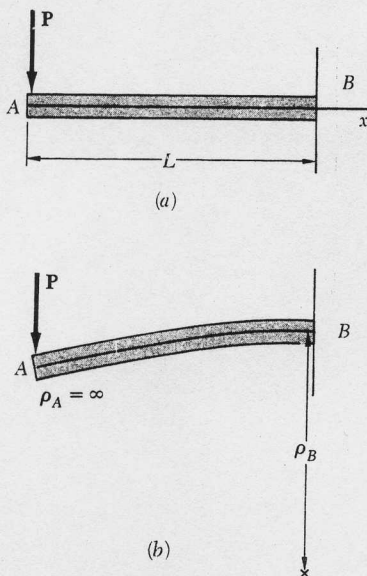


Fig. 8.1

