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Modeling and analysis of micro piezoelectric power generators for micro-electromechanical-systems applications

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Abstract

The extremely small size of the micro-electromechanical systems (MEMS) makes them widely suitable for some special applications. The simplicity of the piezoelectric micro-generators is attractive for MEMS applications, especially for remote systems. In this paper, a general concept of the piezoelectric energy conversion is first given. A simple design modeling and analysis of the '31' transverse mode type piezoelectric micro-generator is presented. The output power is taken as the indicated parameters for the generator. The energy conversion efficiency of the generator, which is dependent on the operation frequency, is expressed in the frequency domain. A case study of laminated type micro-generators using PZT-PIC 255 for MEMS applications is given and the use of single crystal PZN-8% PT is also studied for comparison. Some design guidelines are presented based on the simulation results.

1. Introduction

As micro-electromechanical systems (MEMS) and smart technologies mature, the remote systems and embedded structures attract more and more interest. In applications such as micro-airplanes, the system needs an independent power supply. Traditional batteries are usually used as the source of electric energy. The mass to electrical power ratio is quite high for batteries. Advances in battery technology have not matched the rapid advances in integrated circuit technology. Developing a miniature self-contained renewable power supply is an alternative solution for applications of remote micro-systems.

The energy conversion of light, thermal or mechanical sources is an important aspect for power generators. Piezoelectric materials are prospective materials for energy conversion since they have good electrical–mechanical

coupling effects. As well as the solar generator and electromagnetic generator [1], the piezoelectric micro-generator is an alternative for small-sized equipment applications, especially for dynamical systems involving mechanical vibration. In addition, the simplicity of the piezoelectric micro-generator is particularly attractive for use in MEMS.

So far, there have been relatively few reported studies on the piezoelectric power generators for MEMS applications. A small number of experimental studies on electrical power generators using the piezoelectric materials have been reported in the last few years [2–4]. White *et al* [2] presented an approach to design and model the vibration generator, in which a thick-film piezoelectric generator was described and the corresponding experiment was carried out. In that technical note, the micro-generator with piezoelectric materials is a resonant mechanical structure based on a cantilever beam design and seismic mass, and piezoelectric ceramic PZT 5H is used. The maximum output power, however, is about $2 \mu\text{W}$, which is too small for actual applications. A higher power

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output and higher energy conversion efficiency for generators is desired. Ramsay and Clarrk [6] presented a simple design and analysis using a piezoelectric material membrane as power supply for an *in vivo* MEMS application.

The use of piezoelectric micro-generators for MEMS is seldom reported in comparison to the other applications of piezoelectric materials, such as sensors and actuators. In this paper, the general concept of the piezoelectric generator is first presented. The energy flow during the energy conversion is analyzed. The coupling modeling of the piezoelectric micro-generator is presented. Based on the coupling mode, the performance of the generator is analyzed in the frequency domain. An example of the cantilever beam type micro-generator is examined. The use of piezoelectric material PZT-PIC255 and single crystal PZN-8% PT is examined for the case studies. The numerical simulation gives some guidelines for the optimal design for this kind of micro-generator.

2. General concept of piezoelectric energy conversion

Among the several energy conversion materials, piezoelectric materials are widely used for smart structures, normally classified into two different types based on the energy conversion direction. The first one is the actuator type, in which the piezoelectric element undergoes a dimension change when an electric field is applied. The electric energy is converted into mechanical energy based on the indirect piezoelectric effect. The second type is called the sensor type, in which an electric charge is produced when a mechanical stress is applied. The micro-generator is based on the mechanical-to-electrical energy conversion (the so called direct piezoelectric effect).

There are two commonly used coupling modes for piezoelectric power generators, identified by the direction of the mechanical force and electric charge. The direction of polarization is conventionally denoted as the ‘3’ direction. As shown in figure 1(a), the ‘33’ mode implies that charges are collected on the electrode surface perpendicular to the polarization direction when tensile or compressive mechanical forces are applied along the polarization axis. As shown in figure 1(b), the ‘31’ mode implies that charges are collected on the electrode surface perpendicular to the polarization direction when force is applied along the direction perpendicular to the polarization axis [5]. For most piezoelectric materials, the coupling factor of the 33-mode, k_{33} , is larger than the coupling factor of 31-mode, k_{31} . In the 31-mode, the mechanical stresses are applied along the 1-axis. The stresses can be easily achieved by bonding the piezoelectric element to a substructure undergoing bending. The 33-mode energy conversion can achieve higher output power by increasing the layer of the ceramic (Stock type). For very low-pressure source and limited size, the 31-mode conversion may have a greater advantage in energy conversion [6]. For application of the MEMS structures, the size of the generator is small and the environmental sources for mechanical energy are limited. The 31-mode energy conversion is suitable for piezoelectric micro-generators used in MEMS structures.

As shown in figure 2, the energy flow of the piezoelectric micro-generator consists of two energy transformation steps.

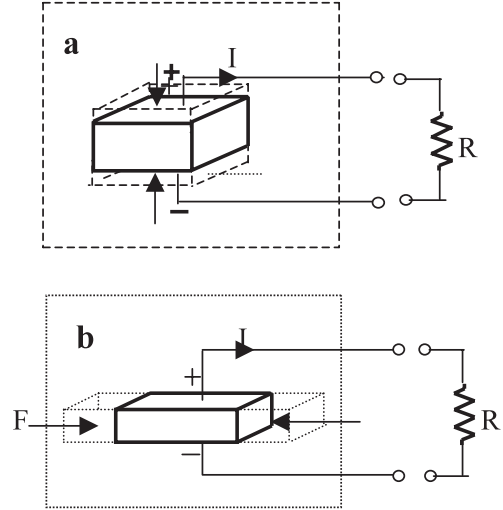


Figure 1. Two coupling modes for piezoelectric materials. (a): ‘33’ coupling mode, (b): ‘31’ coupling mode.

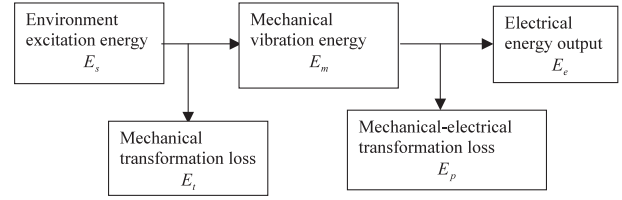


Figure 2. Energy flow chart of the piezoelectric micro-generator.

Firstly, the environmental energy is transformed to the mechanical vibration energy of the seismic mass. In the second step the mechanical vibration energy is transformed into electrical energy, in which the direct piezoelectric effect of the piezoelectric materials is used. The mechanical vibration energy stored in the mechanical system E_m can be divided into two parts:

$$E_e = E_m - E_p = E_m \eta_{me} \quad (1)$$

where E_e and E_p are the electric energy generated by the piezoelectric element and the energy lost during the mechanical–electrical energy conversion. The corresponding piezoelectric element conversion efficiency is defined as

$$\eta_{me} = \frac{E_e}{E_m}. \quad (2)$$

The conversion efficiency from excitation source to kinetic energy is defined as

$$\eta_m = \frac{E_m}{E_s}. \quad (3)$$

An optimal design of the mechanical transforming system can improve the η_m . To increase the conversion efficiency at the piezoelectric part η_{me} , a higher coupling factor and low Q factor and smaller internal damping are desired.

3. Analysis of piezoelectric generators

3.1. Coupling equation for piezoelectric generators

In order to produce the maximum deformation of the piezoelectric materials, and consequently to induce maximum

electric energy output, the piezoelectric generator operates at the resonance frequency of the vibrational structure. The maximum power is generated at the resonance frequency of the generator. Near the resonance frequency, the mechanical behavior of the structure is well described by a single degree-of-freedom (dof) system. In order to simplify the problem, the single modal vibration of the mechanical structure is used to investigate the performance of the system. For a one-dof system generator, such as a piezoelectric laminated beam vibrating under single bending modal, the mechanical-electrical coupling equations can be expressed as follows [7]:

$$m\ddot{\theta} + c\dot{\theta} + k\theta - k_{me}v = f(t) \quad (4)$$

$$q - k_{me}\theta - \frac{1}{c_p}v = 0 \quad (5)$$

where m , c , k are modal mechanical mass, modal mechanical damping coefficient and modal mechanical stiffness respectively. k_{me} is the modal piezoelectric coupling stiffness, $f(t)$ is the modal external mechanical force, θ is the modal displacement, v is the difference in electric potential between the electrodes, q is the electric charge on the electrodes, and c_p is the piezoelectric material inherent capacitance. The coupling stiffness and the inherent capacitance are functions of the mechanical design parameters as well as the piezoelectric material properties. From equations (4) and (5) it can be seen that, due to the coupling between the mechanical and electrical components, the displacement in the piezoelectric materials induces the electrical charge on the electrodes. For application of the piezoelectric power generators, the coupling parameter k_{me} is quite important for properties of the generators. In general, a larger coupling coefficient gives a higher conversion efficiency. In this respect, single piezoelectric crystals are promising for producing high coupling coefficient.

Under certain external excitations, the vibration amplitude of the generator element will be affected by the external electrical load, which is driven by the electrical power induced from the piezoelectric generator. The general equation for output voltage and the charge can be expressed as follows:

$$F(v) + G(q) = 0 \quad (6)$$

where $F(*)$ and $G(*)$ are linear differential operators with respect to time t . For different external loads, the equation will be different. The coupling dynamical equations (4) and (5) describe the relationship between the mechanical variable $\theta(t)$ and electrical variable $q(t)$. The shunting circuit equation (6) gives the condition of the electric load. When the external impedance is a pure resistance R , the circuit diagram is as shown in figure 1(b). Besides the external resistance, there is internal resistance, which is parallel to the external resistance. In this paper, the contribution of the internal resistance for the whole electric closed loop is neglected for the analysis since its value is much larger than the external resistance. Equation (6) can be expressed as

$$R\dot{q} + v = 0. \quad (7)$$

Substituting equation (7) into (5), the following coupling equations can be obtained:

$$m\ddot{\theta} + c\dot{\theta} + \left(k + \frac{k_{me}^2}{c_p}\right)\theta - \frac{k_{me}}{c_p}q = f(t) \quad (8a)$$

$$R\dot{q} + \frac{1}{c_p}q - \frac{k_{me}}{c_p}\theta = 0. \quad (8b)$$

3.2. Conversion efficiency of piezoelectric generators

The transfer function between the electric charge and mechanical force can be expressed as

$$H_{qf}(\omega) = \frac{q(\omega)}{f(\omega)} = \frac{k_{me}}{(-m\omega^2 + j\omega c + k)(1 + jR\omega c_p) + jk_{me}^2 R\omega} \quad (9)$$

$$H_{xf}(\omega) = \frac{\theta(\omega)}{f(\omega)} = \frac{(jc_p \omega R + 1)}{(-m\omega^2 + j\omega c + k)(1 + jR\omega c_p) + jk_{me}^2 R\omega}. \quad (10)$$

The amplitude of the instantaneous power dissipated due to the structural damping within the vibration system can be expressed as [8]

$$\frac{dE_p}{dt} = \frac{d(c \cdot \dot{\theta} \cdot \theta)}{dt} = c\omega^2 |\theta|^2 = c\omega^2 |f H_{xf}|^2. \quad (11)$$

The amplitude of the output electric instantaneous power can be expressed as the power flow through the external resistance:

$$\frac{dE_e}{dt} = R \cdot \dot{q} \cdot q = R\omega^2 |q|^2 = R\omega^2 |f H_{qf}|^2 \quad (12)$$

where $|\cdot|$ means the absolute value of the complex number.

When the external excitation is harmonic, the time-averaged power is half of the amplitude of the instantaneous power. The mechanical vibration energy is dissipated due to the external resistance and the internal mechanical damping. The conversion efficiency of energy is the same as the time-averaged power ratio, which can be expressed as a function of the vibration frequency as follows:

$$\eta_{me} = \frac{R|f H_{qf}|^2}{R|f H_{df}|^2 + c|f H_{xf}|^2}. \quad (13)$$

Substituting equations (9) and (10) into (13), the conversion efficiency under harmonic condition can be rewritten as

$$\eta_{me} = \frac{Rk_{me}}{Rk_{me} + c + c\omega^2 c_p^2 R^2}. \quad (14)$$

From the equation, it can be seen that as the frequency increases, the conversion efficiency is reduced, though the output power increases. Increasing the vibration frequency can increase the output power and is desirable for micro-generators [3]. However, this is based on the assumption that the environmental energy source is infinite. Since for most MEMS applications the energy sources are limited, higher conversion efficiency should be emphasized during the design stage of the piezoelectric micro-generators.

From equation (14), it can be seen that reducing the mechanical damping can decrease the energy dissipated to thermal energy. The conversion efficiency is proportional to the coupling parameter k_{me} .

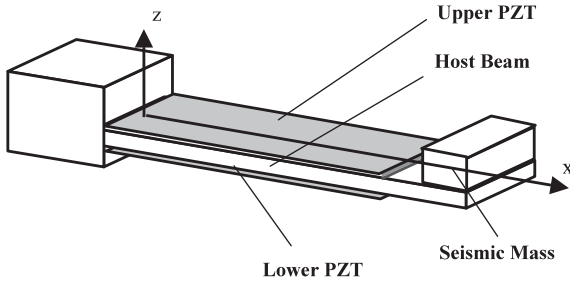


Figure 3. A prototype of the piezoelectric laminated generator.

4. Piezoelectric laminated type generators

4.1. Design parameters of laminated generators

It is easy to achieve a certain amount of stress with the '31'-mode by bonding the piezoelectric element to a substructure undergoing bending. In comparison with '33'-mode generators, this is more suitable for MEMS applications. A typical 31-mode power generator is a beam type piezoelectric power generator, in which a piezoelectric layer is bonded to a host element. When the host element is vibrating under the external excitation, a corresponding deformation is induced in the piezoelectric layer. A prototype of a piezoelectric laminated beam generator is shown in figure 3. A seismic mass is attached to the end of the beam to adjust the resonance frequency of the system.

In order to simplify the analysis, the generator is regarded as a cantilever beam with uniform thickness. Based on the elastic theory of a beam, the displacement in the x -direction for a point within the beam can be expressed as [9]

$$u_x = u_0(x, t) - z \frac{\partial w(x, t)}{\partial x} \quad (15)$$

where u_0 is the axial displacement, which is neglected in this case, w is the transverse displacement of the beam and z is the distance from the neutral plane. The strain along the x -direction is expressed as

$$\varepsilon_x = \frac{\partial u_x}{\partial x} = -z \frac{\partial^2 w}{\partial x^2}. \quad (16)$$

Assume that the thin layer of the piezoelectric added to the surface of the beam does not change the deformed shape of the beam, and only increases the equivalent bending stiffness. Consequently, the amplitude of the vibration under the same excitation is reduced in comparison to the same beam without the piezoelectric layer. The transverse displacement of the beam is controlled by the external mechanical excitation source.

The linear piezoelectric theory is employed. The charge in the x -direction is assumed to be zero, $D_x = 0$. The stress in the z -direction is assumed to be zero within the piezoelectric layer, $\sigma_z|_{z < \Delta} = 0$. This assumption is valid when the piezoelectric layer thickness in comparison to the length of the beam is very thin. The electrical displacement in the z -direction within the piezoelectric layer is expressed as a function of the strain in the x -direction and the electric field within the piezoelectric layer:

$$D_z = e_{31}\varepsilon_x + \epsilon_{33}E_z \quad (17)$$

where e_{31} is the piezoelectric constant in the 31 coupling direction, ϵ_{33} is the dielectric constant and E_z is the electric field in the z -direction within the piezoelectric layer. The charge collected on the electrode surface can be expressed as the electrical displacement integral on the area of the surface:

$$Q = \int_A D_z dA = b \int_{l_0}^{l_1} (e_{31}\varepsilon_x + \epsilon_{33}E_z) dx \quad (18)$$

where E_z is the electric field within the piezoelectric layer and b is the width of the beam (assumed to be constant along the whole range). Because there is an electrode covered on the surface, the potential of the surface is the same. Assuming that the potential difference between the upper surface and lower surface of the piezoelectric layer is denoted as v , under the uniform electrical field assumption, the electric field can be approximately expressed as

$$E_z = -\frac{\partial v}{\partial z} = -\frac{v}{\Delta} \quad (19)$$

where Δ is the thickness of the piezoelectric layer. For simplicity the thickness of the layer is assumed to be the same within the whole coverage area. Substituting equation (19) into (18),

$$\begin{aligned} Q &= -b \int_{l_0}^{l_1} e_{31} \frac{h}{2} \frac{\partial^2 w}{\partial x^2} dx + b \int_{l_0}^{l_1} \epsilon_{33} E_z dz \\ &= \frac{bhe_{31}}{2} [\varphi(l_0) - \varphi(l_1)] - bL\epsilon_{33} \frac{v}{\Delta} \end{aligned} \quad (20)$$

where $\varphi(x, t) = \frac{\partial w(x, t)}{\partial x}$ is the flexibility of the beam, $\varphi(l_0)$ and $\varphi(l_1)$ are the corresponding values at the beginning of the piezoelectric layer and that at the end of the piezoelectric layer, h is thickness of host beam and L is the length of the piezoelectric layer on the beam surface.

It should be emphasized that the current, charge and voltage are all functions of the time. The frequency of these period functions is dependent upon the mechanical vibration. Because the differential of the charge on the electrode surface is the current flow out to the external impedance, the amplitude of the current is that of the charge times the frequency:

$$I = \omega Q. \quad (21)$$

The relationship between voltage and current for an electrical circuit with pure resistance is expressed as

$$I = \frac{v}{R} \quad (22)$$

and the voltage has the same phase as the current. Combining equations (20)–(22), the amplitude of the current can be determined as

$$I_m = \frac{\omega b h e_{31} [\varphi(l_0) - \varphi(l_1)]}{2(1 + bL\epsilon_{33} \frac{\omega R}{\Delta})} R. \quad (23)$$

Since the charge Q on the electrode surface is connected to the external impedance, if the external impedance is a pure resistance, the output voltage and the current have the same phase. The output power can be expressed as

$$P = Iv = I^2 R. \quad (24)$$

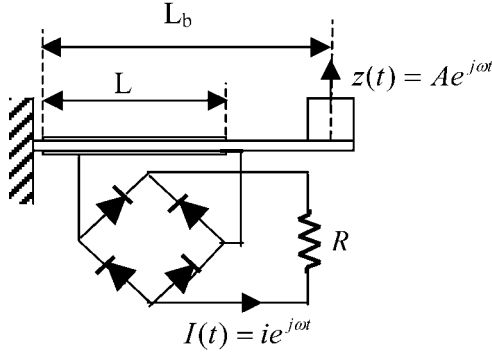


Figure 4. Schematic diagram of the generator.

The steady-state condition with a harmonic excitation force, the output current and voltage are harmonic functions too. For an AC electrical circuit, substituting equation (23) into (24), the time-average of the output power of one piezoelectric layer can be expressed by multiplication of the voltage and current as

$$\bar{P} = \frac{v_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = \frac{\omega^2 b^2 h^2 e_{31}^2 [\varphi(l_0) - \varphi(l_1)]^2}{8(1 + bL\epsilon_{33} \frac{\omega R}{\Delta})^2} R \quad (25)$$

where \bar{P} is the time-averaged output power. With two layers attached to the host beam as shown in figure 3, the total average power is two times that of power generated by a single piezoelectric layer.

For a cantilever beam under a certain vibration mode, the vibration states can be determined using the separation of variables from elastic theory [10]:

$$w(x, t) = W(x)e^{j\omega t} \quad (26)$$

where $W(x)$ is shape function along the cantilever beam, which can be obtained from the boundary conditions [10]:

$$W(x) = C_1 \left\{ [\cosh(\beta x) - \cos(\beta x)] + \frac{\cosh(\beta L_b) + \cos(\beta L_b)}{-\sinh(\beta L_b) - \sin(\beta L_b)} [\sinh(\beta x) - \sin(\beta x)] \right\} \quad (27)$$

where L_b is the length of the beam as indicated in figure 4. For convenience, the vibration state of the beam at the point where the seismic mass is attached, which is measured as $Ae^{j\omega t}$ as shown in figure 4, is taken as the system indication to determine the vibration amplitude. The constant C_1 in the equation can be expressed by the vibration amplitude of the seismic mass at the end of the beam:

$$W(x)|_{x=L_b} = A \quad (28)$$

where A is the vibration amplitude measured at the end of the beam. Description of the bending flexibility value can be obtained using a separation of variables:

$$\varphi(l_0) - \varphi(l_1) = (W'(l_0) - W'(l_1))e^{j\omega t} = \bar{A}e^{j\omega t}. \quad (29)$$

Substituting equations (29) into (25), the time-averaged power is rewritten as

$$\bar{P} = vI = \frac{\omega^2 b^2 h^2 e_{31}^2 \bar{A}^2}{4(1 + bL\epsilon_{33} \frac{\omega R}{\Delta})^2} R. \quad (30)$$

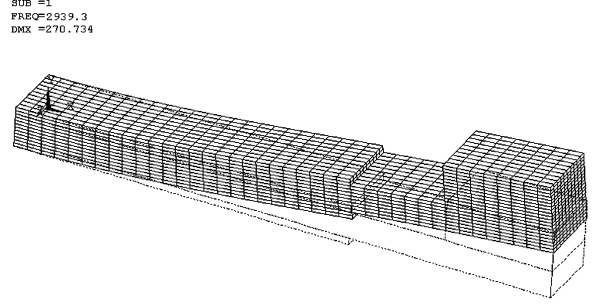


Figure 5. First mode of the piezoelectric laminated generator.

From the above equations, it is found that the output power is dependent upon several parameters. The external resistance R , and frequency of the vibration ω are interesting design factors for piezoelectric generators. In the next section, detailed discussion will be given about the performance of the output power as a function of these parameters.

4.2. Case study and analysis

The estimated evaluation of a typical piezoelectric laminated type generator is used to ascertain the performance of the piezoelectric generators. Modified lead zirconate titanate (PZT) provided by PI Ceramic Company is employed for the generator design. The material parameters of the PIC-255 ceramic used for the simulation and design are listed in table 1.

The beam is excited at its first bending vibration mode. The first bending vibration of the generator is calculated using ANSYS. The frequency is determined at 2939.8 Hz as shown in figure 5, which can be adjusted by the seismic mass added to the end of the beam. Assume that the thin layer of the piezoelectric added on the surface of the beam does not change the deformation shape of the beam, and only increases the equivalent bending stiffness. Consequently, the amplitude of the vibration under the same excitation is reduced in comparison to the same beam without the piezoelectric layer. The transverse displacement of the beam is controlled by the external mechanical excitation source.

For a typical application, a continuous supply of power required generator output power of 0.1 mW at least and intermittent power supply required about 1 μ W [6]. Table 2 lists the typical output powers under different vibration amplitude, in which the external resistance is set as $R = 40\,000 \Omega$. To achieve the power requirement for continuous usage using material PZT-PIC255, a vibration amplitude measured on the seismic mass of 15 μ m is required. For intermittent usage, the corresponding amplitude can be lower than 5 μ m. Under the same conditions, the single crystal PZN-8% PT, whose properties from [11] are shown in table 1, is used for comparison. It is found that it does not give better results at the lower operational frequency. The output power is smaller than that of PZT-PIC255. To achieve the power requirement for continuous usage, a vibration amplitude on the seismic mass of 20 μ m is required. However, the output power using PZN-8% PT is much higher as the operational frequency increases (as shown in figure 6). This is because the output power is a function of operation frequency as expressed in equation (30). The order of the operation frequency in the dominator and the

Table 1. Parameters of the piezoelectric laminated generator.

Structure parameters	PZT-PIC255	PZN-8% PT
Thickness of piezo-layer, $\Delta = 0.1$ mm	Piezoelectric constant,	Piezoelectric constant
Width of piezo-layer, $b = 1$ mm	$e_{31} = 11.2$ C m ⁻²	$e_{31} = 5.8$ C m ⁻²
Thickness of the beam, $h = 0.4$ mm	Relative permittivity,	Relative permittivity,
Length of the piezo-layer, $L = 5$ mm	$\epsilon_{33}/\epsilon_0 = 1800$	$\epsilon_{33}/\epsilon_0 = 560$
	Absolute permittivity, $\epsilon_0 = 8.85 \times 10^{-12}$ F m ⁻¹	

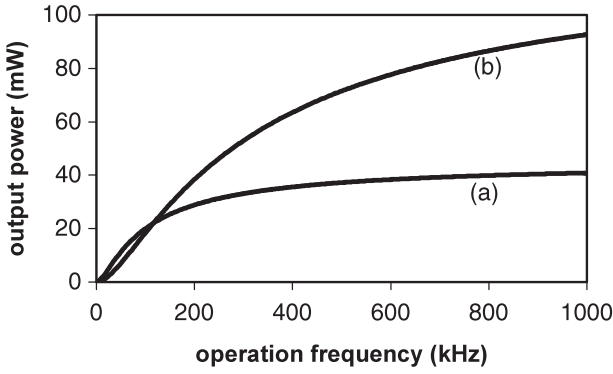


Figure 6. Output power as a function of vibration frequency: (a) material PZT PI515; (b) material PZN-8% PT.

Table 2. Typical output powers with different vibration amplitudes frequency 2939 Hz.

Output power (mW)		Amplitude on seismic mass (μ m)	Coefficients \bar{A}
PZT-PIC 255	PZN-8% PT		
0.64	0.31	30	2.78×10^{-3}
0.44	0.21	25	2.3×10^{-3}
0.28	0.14	20	1.8×10^{-3}
0.16	0.077	15	1.4×10^{-3}
0.07	0.034	10	1×10^{-3}
0.016	0.008	5	0.46×10^{-3}

numerator are the same. PZN-8% PT has larger ϵ_{33} , which is in the coefficient of the dominator and PZT-PIC255 has larger e_{31} which is in the coefficient of the numerator of the expression. With lower operation frequency, the smaller ϵ_{33} contributes more to output power, and with high operation frequency, the larger e_{31}/ϵ_{33} ratio contributes more to output power.

The power generated is proportional to the vibration amplitude, as expected. Increasing the vibration amplitude can obviously improve the output power. In addition, the external resistance affects the output power. There is an optimal resistance that gives the maximum output power. The output power as a function of the external resistance is shown in figure 7, in which the vibration amplitude is set as a constant. There is an optimal external resistance that gives the maximum output power, which is in agreement with the experimental results reported by White [2]. It is shown that the external resistance of 68 k Ω gives the maximum output power of 0.66 mW for generator using PZT-PIC255, while for PZN-8% PT the maximum output power 0.571 mW exists at an external resistance value of 215 k Ω . Comparing output power of the generators using PZT-PIC255 and PZN-8% PT as shown in figures 6 and 7, it is found that PZT-PIC255 is more sensitive

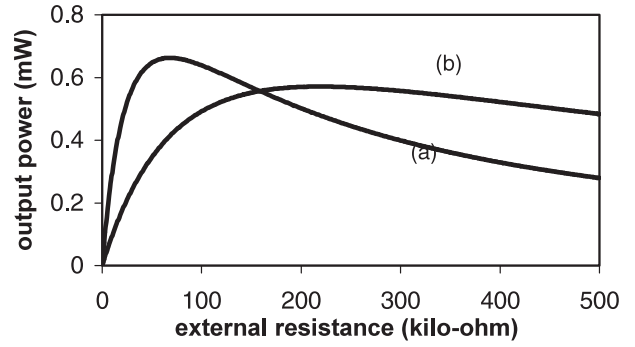


Figure 7. Output power as a function of the external resistance at 2939.0 Hz. (a) PZT PI515, (b) PZN-8% PT.

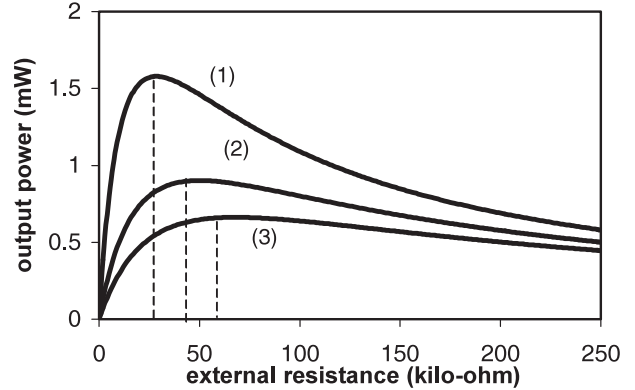


Figure 8. Output power as a function of external resistance with different operational frequencies using PZT-PIC255: (1) vibration frequency 7000 Hz; (2) 4000 Hz; (3) 2939.0 Hz.

to the external resistance, while PZN-8% PT is more sensitive to the operational frequency.

The resistance, which gives the maximum output power, is dependent on the variables of the system. Conversely, when the resistance requirement is determined by the application, the corresponding parameters of the system can be designed to achieve the maximum output power. As the frequency of the vibration increases, the corresponding optimal resistance is reduced. Figure 8 gives the output powers of generators using PZT-PIC255 as a function of the external resistance with different vibration frequencies. As the operational frequency increases, the output power increases and the optimal point shifts to the left.

From equation (30), the output power reaches its maximum value when the external resistance is

$$R_p^* = \frac{\Delta}{bL\epsilon_{33}\omega}. \quad (31)$$

The energy conversion efficiency is also a function of the external resistance as indicated in equation (14). The maximum conversion efficiency can be obtained at a certain external resistance. The external resistance that gives the maximum conversion efficiency can be obtained from equation (14):

$$R_{\eta}^* = \frac{1}{c_p \omega}, \quad (32)$$

$R_{\eta}^* = R_p^*$. For piezoelectric laminated beam-type generators, the capacitance value of the piezoelectric layer can be expressed as $c_p = \frac{bL\epsilon_{33}}{\Delta}$.

The maximum energy conversion efficiency can be expressed by substituting equation (32) into (14):

$$\eta_{\max} = \frac{k_{\text{me}}}{k_{\text{me}} + 2c_p c \omega}. \quad (33)$$

The maximum conversion efficiency is inversely proportional to the vibration frequency and mechanical damping. As expected, increasing the coupling stiffness can improve the conversion efficiency.

The frequency of the vibration is an important factor that affects the output performance. From the equation of the power as a function of the frequency, increasing the mechanical vibration frequency can increase the output power. However, when the frequency reaches a certain value, the output power will increase very gradually as the frequency continues to increase further, as shown in figure 6. The value of this certain frequency depends on the design parameters of the system. The design of the structure can increase this limiting frequency. In addition, as the frequency increases, the deformation amplitude is decreased. This will also reduce the output power. After a certain frequency, the process of solely increasing the vibration frequency will not improve the output power and, on the contrary, may even reduce the output power.

The conversion efficiency from kinetic energy to electric energy is dependent on the structural damping of the system (including the environment air flow damping and fluid flow damping when used in a fluid environment, for example for application in body bio-MEMS). In addition, the conversion efficiency is inversely proportional to the operational frequency, as indicated in equation (14). Increasing the frequency of the vibration can improve the output power. At the same time, however, the energy dissipated due to damping is increased. This consideration should be important in conditions where the environmental energy source is limited.

In order to improve the conversion efficiency, reducing the structural damping is desirable. In principle, from the vibration theory, the amplitude of the vibration will be infinite and the kinetic energy of the vibration will be totally converted to electrical energy, when the damping factor is zero. But in practice this is not possible. Besides the power conversion efficiency, a certain value of damping is needed to keep the amplitude of the vibration within a certain range, since for MEMS applications there is not much space for a larger displacement of the seismic mass.

5. Conclusion

This paper gives a simple model for the analysis of piezoelectric power generator application in MEMS. The output power and the conversion efficiency are obtained, in which the mechanical–electrical coupling effects are included. The output power is used to evaluate the performance of the generators. Some interesting aspects that affect the output power are discussed. From this analysis, it is found that there is an optimal external resistance that gives the maximum output power, which is in agreement with the experimental results reported by White [3]. Furthermore, increasing the frequency of the vibration can improve the output power, while beyond a certain value, further improvement cannot be achieved by simply increasing the vibration frequency. At the higher frequency, single crystal PZN-8% PT can achieve much higher output power in comparison to piezoelectric material PZT PIC255. The performance of PZN-8% PT is more sensitive to operational frequency and that of PZT-PIC255 is more sensitive to external resistance.

For MEMS applications, the size of the generators is small, of the order of several millimetres, and the vibration amplitude is limited by its operation environment. It is a challenge to increase the deformation of the piezoelectric element under a limited excitation. Within a certain range, increasing the frequency can increase the output power dramatically. But at the same time, due to the inertia effect, it will be more difficult to maintain the same vibration amplitude as the frequency increases.

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